$$\frac{dy(x)}{dx} = \sin\left(\frac{y(x)}{x}\right) + \frac{y(x)}{x}$$

Let $y(x) = x\,v(x)$, which gives $\frac{dy(x)}{dx} = x\,\frac{dv(x)}{dx} + v(x)$:

$$x\,\frac{dv(x)}{dx} + v(x) = \sin(v(x)) + v(x)$$

Solve for $\frac{dv(x)}{dx}$:

$$\frac{dv(x)}{dx} = \frac{\sin(v(x))}{x}$$

Divide both sides by $\sin(v(x))$:

$$\csc(v(x))\,\frac{dv(x)}{dx} = \frac{1}{x}$$

Integrate both sides with respect to x:

$$\int \csc(v(x))\,\frac{dv(x)}{dx}\,dx = \int \frac{1}{x}\,dx$$

Evaluate the integrals:

$$-\log\left(\cos\left(\frac{v(x)}{2}\right)\right) + \log\left(\sin\left(\frac{v(x)}{2}\right)\right) =$$

$\log(x) + c_1$, where c_1 is an arbitrary constant.

Solve for $v(x)$:

$$v(x) = 2\cot^{-1}\left(\frac{e^{-c_1}}{x}\right)$$

Simplify the arbitrary constant:

$$v(x) = 2\cot^{-1}\left(\frac{c_1}{x}\right)$$

Substitute back for $y(x) = x\,v(x)$:

$$y(x) = 2\,x\cot^{-1}\left(\frac{c_1}{x}\right)$$