**Question 1.** *Use mathematical induction to show that* $n! \geq 2^{n-1}$ *for* $n = 1, 2, \dots$.

*Solution.* The base of induction: if $n = 1$, then $n! = 1! = 1$ and $2^{n-1} = 2^{1-1} = 2^0 = 1$, so $n! = 2^{n-1}$ in this case.

The induction step: suppose $n! \geq 2^{n-1}$ for some $n \geq 1$. Prove that $(n+1)! \geq 2^{(n+1)-1}$. Indeed,

$$(n+1)! = n! \cdot (n+1) \geq 2^{n-1} \cdot 2 = 2^n = 2^{(n+1)-1},$$

as desired. □