Question #12765

How to integrate $\int \sqrt{tanx}\,dx$.

Solution.

Make substitution $y = \sqrt{tanx}$, then our integral obviously becomes $\int \frac{2y^2}{1+y^4}\,dy = \int \frac{1+1/y^2}{y^2+1/y^2}\,dy + \int \frac{1-1/y^2}{y^2+1/y^2}\,dy = 1/\sqrt{2}tan^{-1}(\frac{y^2-1}{\sqrt{2}y}) + \frac{1}{2\sqrt{2}}\log|\frac{y^2-\sqrt{2}y+1}{y^2+\sqrt{2}y+1}|$.
Substitution $y = tanx$ leads us to the desired result.