Solve the separable equation $\frac{dy(x)}{dx} = (y(x) - 1)\, y(x)$:

Divide both sides by $(y(x) - 1)\, y(x)$:

$$\frac{\frac{dy(x)}{dx}}{(y(x)-1)\, y(x)} = 1$$

Integrate both sides with respect to x:

$$\int \frac{\frac{dy(x)}{dx}}{(y(x)-1)\, y(x)}\, dx = \int 1\, dx$$

Evaluate the integrals:

$\log(-y(x) + 1) - \log(y(x)) = x + c_1$, where c_1 is an arbitrary constant.

Solve for $y(x)$:

$$y(x) = \frac{1}{e^{x+c_1} + 1}$$

Simplify the arbitrary constant:

$$y(x) = \frac{1}{c_1\, e^x + 1}$$