

$$f \in \mathbb{Z}_7[x], f(x) = ax^4 + bx^3 + cx^2 + dx + e$$

$$f(0) = 1 \Rightarrow e = 1$$

$$f(1) = 4 \Rightarrow a + b + c + d = 3$$

$$f(2) = 6 \Rightarrow 16a + 8b + 4c + 2d = 5 \Rightarrow 2a + b - 3c + 2d = -2$$

$$f(4) = 3 \Rightarrow 16^2a + 64b + 16c + 4d = 2 \Rightarrow -3a + b + 2c - 3d = 2$$

Suppose,  $a = 1$

$$\begin{cases} b + c + d = 3 \\ b - 3c + 2d = 3 \\ b + 2c - 3d = -2 \end{cases} \Rightarrow 3b = 4 \Rightarrow b = 6 = -1 \Rightarrow \begin{cases} c + d = 4 \\ 2c - 3d = -1 \end{cases} \Rightarrow \begin{cases} 2c + 2d = 1 \\ 2c - 3d = -1 \end{cases}$$

$$5d = 2 \Rightarrow d = 6 = -1 \Rightarrow c = 5 = -2$$

$$f(x) = x^4 - x^3 - 2x^2 - x + 1$$