All residues are in poles of denominator, other words roots of $z^4+1=0$.
They are

$$z_1=e^{i\pi/4}$$

$$z_2=e^{3i\pi/4}$$

$$z_3=e^{-3i\pi/4}$$

$$z_4=e^{-i\pi/4}$$

Then:

$$res\left[f\left(z_1\right)\right]=\frac{1}{4z^3}\Big|_{z=z_1}=\frac{1}{4}e^{-3i\pi/4}$$

$$res\left[f\left(z_2\right)\right]=\frac{1}{4z^3}\Big|_{z=z_2}=\frac{1}{4}e^{-9i\pi/4}$$

$$res\left[f\left(z_3\right)\right]=\frac{1}{4z^3}\Big|_{z=z_3}=\frac{1}{4}e^{9i\pi/4}$$

$$res\left[f\left(z_4\right)\right]=\frac{1}{4z^3}\Big|_{z=z_4}=\frac{1}{4}e^{3i\pi/4}$$