1)

$$\cot\theta + \frac{3}{sin\theta} + 3 = 0$$

$$cos\theta + 3 = -3sin\theta$$

$$cos\theta + 3sin\theta + 3 = 0$$

Results:

$$\theta = 2\pi n - 2\tan^{-1}(2) \text{ and } n \in \mathbb{Z}$$

$$\theta = \frac{1}{2}\pi(4n-1) \text{ and } n \in \mathbb{Z}$$

Possible intermediate steps:

$$3\sin(\theta) + \cos(\theta) + 3 = 0$$

Substitute $u = \tan\left(\frac{\theta}{2}\right)$. Then $\sin(\theta) = \frac{2u}{u^2+1}$ and $\cos(\theta) = \frac{1-u^2}{u^2+1}$:

$$\frac{6u}{u^2+1} + \frac{1-u^2}{u^2+1} + 3 = 0$$

Expand out terms of the left hand side:

$$-\frac{u^2}{u^2+1} + \frac{6u}{u^2+1} + \frac{1}{u^2+1} + 3 = 0$$

Write the left hand side as a single fraction:

$$\frac{2(u^2+3u+2)}{u^2+1} = 0$$

Multiply both sides by $u^2 + 1$:

$$2(u^2 + 3u + 2) = 0$$

Divide both sides by 2:

$$u^2 + 3u + 2 = 0$$

Subtract 2 from both sides:

$$u^2 + 3u = -2$$

Add $\frac{9}{4}$ to both sides:

$u^2 + 3u + \frac{9}{4} = \frac{1}{4}$

Factor the left hand side:

$\left(u + \frac{3}{2}\right)^2 = \frac{1}{4}$

Take the square root of both sides:

$\left|u + \frac{3}{2}\right| = \frac{1}{2}$

Eliminate the absolute value:

$u + \frac{3}{2} = -\frac{1}{2}$ or $u + \frac{3}{2} = \frac{1}{2}$

Subtract $\frac{3}{2}$ from both sides:

$u = -2$ or $u + \frac{3}{2} = \frac{1}{2}$

Substitute back for $u = \tan\left(\frac{\theta}{2}\right)$:

$\tan\left(\frac{\theta}{2}\right) = -2$ or $u + \frac{3}{2} = \frac{1}{2}$

Take the inverse tangent of both sides:

$\frac{\theta}{2} = -\tan^{-1}(2)$ or $u + \frac{3}{2} = \frac{1}{2}$

Divide both sides by $\frac{1}{2}$:

$\theta = -2\tan^{-1}(2)$ or $u + \frac{3}{2} = \frac{1}{2}$

Subtract $\frac{3}{2}$ from both sides:

$\theta = -2\tan^{-1}(2)$ or $u = -1$

Substitute back for $u = \tan\left(\frac{\theta}{2}\right)$:

$\theta = -2\tan^{-1}(2)$ or $\tan\left(\frac{\theta}{2}\right) = -1$

Take the inverse tangent of both sides:

$\theta = -2\tan^{-1}(2)$ or $\frac{\theta}{2} = -\frac{\pi}{4}$

Divide both sides by $\frac{1}{2}$:

$\theta = -2\tan^{-1}(2)$ or $\theta = -\frac{\pi}{2}$

2)

$tan\theta + tan2\theta + tan3\theta = tan\theta * tan2\theta * tan3\theta$

$$\tan(\theta) + \frac{2\tan(\theta)}{1-\tan^2(\theta)} + \frac{3\tan(\theta)-\tan^3(\theta)}{1-3\tan^2(\theta)} = \tan(\theta)\times\frac{2\tan(\theta)}{1-\tan^2(\theta)}\times\frac{3\tan(\theta)-\tan^3(\theta)}{1-3\tan^2(\theta)}$$

Results:

$\theta = \frac{2\pi n}{3}$ and $n \in \mathbb{Z}$

$\theta = \frac{1}{3}(2\pi n + \pi)$ and $n \in \mathbb{Z}$