

$$\begin{aligned} & \cos^2(x) + \cos^2\left(x + \frac{2\pi}{3}\right) + \cos^2\left(x - \frac{2\pi}{3}\right) = \\ & = \cos^2 x + \frac{1 + \cos\left(2x + \frac{4\pi}{3}\right) + \cos\left(2x - \frac{4\pi}{3}\right) + 1}{2} = \\ & = 1 + \cos^2 x + \cos(2x)\cos\left(\frac{4\pi}{3}\right) = 1 + \frac{1 + \cos 2x}{2} - \frac{\cos 2x}{2} = \frac{3}{2} \end{aligned}$$