$$\cos\frac{3\pi}{8}=\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)=\sin\frac{\pi}{8}$$

$$\cos\frac{7\pi}{8}=-\cos\frac{\pi}{8}$$

$$\cos\frac{5\pi}{8}=-\cos\frac{3\pi}{8}$$

$$\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=$$

$$=\cos^4\frac{\pi}{8}+\sin^4\frac{\pi}{8}+\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}=$$

$$=2\left(\cos^4\frac{\pi}{8}+\sin^4\frac{\pi}{8}\right)=2\left(\left(\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}\right)^2-2\cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right)=$$

$$=2\left(1-\frac{1}{2}\sin^2\frac{\pi}{4}\right)=2-\frac{1}{2}=\frac{3}{2}$$