

$$\begin{aligned} \tan 20 \tan 40 \tan 80 &= \tan 20 \frac{\tan 60 - \tan 20}{1 + \tan 60 \tan 20} \frac{\tan 60 + \tan 20}{1 - \tan 60 \tan 20} = \\ &= \tan 20 \frac{\tan^2 60 - \tan^2 20}{1 - \tan^2 60 \tan^2 20} = \frac{3 \tan 20 - \tan^3 20}{1 - 3 \tan^2 20} \end{aligned}$$

$$\tan 60 = \tan(40 + 20) = \frac{\tan 40 + \tan 20}{1 - \tan 40 \tan 20} = \frac{\frac{2 \tan 20}{1 - \tan^2 20} + \tan 20}{1 - \frac{2 \tan^2 20}{1 - \tan^2 20}} = \frac{3 \tan 20 - \tan^3 20}{1 - 3 \tan^2 20}$$