Prove that

$$sin\alpha * \sin(60-\alpha) * \sin(60+\alpha) = \frac{1}{4}\sin(3\alpha)$$

Note: We will use the known formulas

$$\sin(60 \pm \alpha) = sin60 * cos\alpha \pm cos60 * sin\alpha = \frac{\sqrt{3}}{2} * cos\alpha \pm \frac{1}{2} * sin\alpha$$

$$sin3\alpha = 3sin\alpha - 4sin^3\alpha$$

$$sin^2\alpha + cos^2\alpha = 1$$

Solution:

$sin\alpha * \sin(60-\alpha) * \sin(60+\alpha) =$

$$= sin\alpha * \left(\frac{\sqrt{3}}{2} * cos\alpha - \frac{1}{2} * sin\alpha\right) * \left(\frac{\sqrt{3}}{2} * cos\alpha + \frac{1}{2} * sin\alpha\right)$$

$$= sin\alpha * \left(\frac{3}{4} * cos^2\alpha - \frac{1}{4} * sin^2\alpha\right) = \frac{1}{4} * sin\alpha * (3cos^2\alpha - sin^2\alpha)$$

$$= \frac{1}{4} * sin\alpha * (3(1 - sin^2\alpha) - sin^2\alpha) = \frac{1}{4} * sin\alpha * (3 - 4sin^2\alpha)$$

$$= \frac{1}{4} * (3sin\alpha - 4sin^3\alpha) = \frac{1}{4} * \sin(3\alpha)$$