

$$\tan(60+A) \cdot \tan(60-A) = 2\cos 2A + \frac{1}{2\cos 2A - 1}$$

$$tg(60+A)tg(60-A) = 2\cos 2A + \frac{1}{2\cos 2A - 1}$$

We have

$$tg(60+A) = \frac{tg 60 + tg A}{1 - tg 60 tg A} = \frac{\sqrt{3} + tg A}{1 - \sqrt{3} tg A}$$

$$tg(60-A) = \frac{tg 60 - tg A}{1 + tg 60 tg A} = \frac{\sqrt{3} - tg A}{1 + \sqrt{3} tg A}$$

$$\begin{aligned} tg(60+A)tg(60-A) &= \frac{\sqrt{3} + tg A}{1 - \sqrt{3} tg A} \cdot \frac{\sqrt{3} - tg A}{1 + \sqrt{3} tg A} = \frac{3 - tg^2 A}{1 - 3tg^2 A} = \frac{3 - \frac{\sin^2 A}{\cos^2 A}}{1 - 3\frac{\sin^2 A}{\cos^2 A}} = \\ &= \frac{3\cos^2 A - \sin^2 A}{\cos^2 A - 3\sin^2 A} = \frac{2\cos^2 A + \cos 2A}{\cos 2A - 2\sin^2 A} = \frac{1 + \cos 2A + \cos 2A}{\cos 2A - 1 + \cos 2A} = \frac{1 + 2\cos 2A}{-1 + 2\cos 2A} = \\ &= \frac{1}{2\cos 2A - 1} + \frac{2\cos 2A}{2\cos 2A - 1} \end{aligned}$$

Thus we have

$$tg(60+A)tg(60-A) = \frac{1}{2\cos 2A - 1} + \frac{2\cos 2A}{2\cos 2A - 1}.$$

And identity $tg(60+A)tg(60-A) = 2\cos 2A + \frac{1}{2\cos 2A - 1}$ is wrong, because for $A=60$

$$tg(60+60)tg(60-60) = tg 120 tg 0 = 0 \neq -\frac{3}{2} = 2\cos 120 + \frac{1}{2\cos 120 - 1} = -1 + \frac{1}{-2}$$