Answer on question 36105 – Math – Trigonometry

In triangle ABC, prove that $a\,sin(B-C)+bsin(C-A)+csin(A-B)=0$

Solution

Using the following trigonometric identity

$$\sin(A-B)=\sin A\cos B-\sin B\cos A$$

We get

$$a\,sin(B-C)+bsin(C-A)+csin(A-B)=a\sin B\cos C-a\sin C\cos B+$$

$$+b\sin C\cos A-b\sin A\cos C+c\sin A\cos B-c\sin B\cos A=\cos A\,(b\sin C-c\sin B)+$$

$$+\cos B\,(c\sin A-a\sin C)+\cos C\,(a\sin B-b\sin A)$$

According to the sine theorem we have

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Therefrom

$$b\sin C-c\sin B=c\sin A-a\sin C=a\sin B-b\sin A=0$$

And we get

$$a\,sin(B-C)+bsin(C-A)+csin(A-B)=0.$$

QED