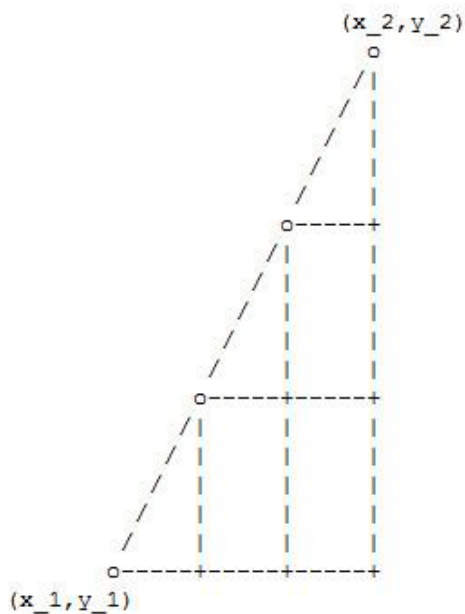**How to find trisecting points?**

This is related to the midpoint formula.

Here is our segment:

```
                    (x_2,y_2)
                        o
                       /|
                      / |
                     /  |
                    /   |
                   /    |
                  o-----+
                 /|     |
                / |     |
               /  |     |
              /   |     |
             o----+-----+
            /|    |     |
           / |    |     |
          /  |    |     |
         /   |    |     |
        o----+----+-----+
    (x_1,y_1)
```

Note that when we divide the segment into three equal parts, we are also dividing the horizontal and vertical legs of this right triangle into three equal parts. (Think similar triangles!)

Now, in terms of the coordinates, how long is that horizontal segment?

$$\frac{x_2 - x_1}{3}$$

How long is that vertical segment?

$$\frac{y_2 - y_1}{3}$$

How much do we have to add to x_1 to find the x-coordinate of each intermediate point? What are those coordinates?

For the first point we need add only one segment. For the second point we need add two segments

So the first point has coordinates $\left(x_1 + \frac{x_2-x_1}{3}, y_1 + \frac{y_2-y_1}{3}\right)$ or it equal $\left(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}\right)$.

The second point has coordinates $\left(x_1 + 2 * \frac{x_2-x_1}{3}, y_1 + 2 * \frac{y_2-y_1}{3}\right)$ or it equal

$$\left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}\right).$$

We find the coordinates of two points

$$p_1\left(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}\right) \qquad p_2\left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}\right).$$

Example

Find the coordinates the points of trisections of the line segment joining the points $A(2,-2)$, $B(-7,4)$.

Solution

The first point has coordinates $p_1 = \left(\frac{2*2-7}{3}, \frac{2*(-2)+4}{3}\right) = (-1,0)$

The second point has coordinates $p_2 = \left(\frac{2+2*(-7)}{3}, \frac{-2+2*4}{3}\right) = (-4,2)$

Answer: $p_1(-1,0)$; $\quad p_2(-4,2)$