Prove that the angle between internal bisector of one base angle and external bisector of other base angle of a triangle equal to one half of the vertical angle

Solution:

Draw a triangle ABC.
Let B and C are base angles and A be the vertical angle.
Draw BX internal bisector of angle B.
Draw CY external bisector of angle C.
Let BX and CY intersect at D.

Now <BDC is the angle between internal bisector of one base angle (B) and external bisector of other base angle (C) of triangle ABC.
From triangle DBC.

$$< BDC = 180 - (< DBC + < DCB)$$

Where <DBC is internally bisected angle of B and <DCB is <C + externally bisected angle of C

$$< BDC = 180 - \left[< \left(\frac{B}{2}\right) + < C + \text{exterior angle of } \frac{C}{2}\right]$$

Recall that exterior angle of C equal $(< A + < B)$
So

$$< BDC = 180 - \left[< \left(\frac{B}{2}\right) + < C + < \left(\frac{A + B}{2}\right)\right]$$

$$< BDC = 180 - \left[< B + < C + < \left(\frac{A}{2}\right)\right]$$

$$< BDC = [180 - (< B + < C)] - < \left(\frac{A}{2}\right)$$

$$< BDC = < A - < \left(\frac{A}{2}\right)$$

$$< BDC = < \left(\frac{A}{2}\right)$$

http://www.AssignmentExpert.com