

$$y'(x) = x z(x) + 1, z'(x) = -x y(x), y(0) = 0, z(0) = 1$$

First-order system of linear differential equations

Solution

$$y(x) = \sqrt{\pi} C\left(\frac{x}{\sqrt{\pi}}\right) \cos\left(\frac{x^2}{2}\right) + \left(\sqrt{\pi} S\left(\frac{x}{\sqrt{\pi}}\right) + 1\right) \sin\left(\frac{x^2}{2}\right)$$

$$z(x) = -\sqrt{\pi} C\left(\frac{x}{\sqrt{\pi}}\right) \sin\left(\frac{x^2}{2}\right) + \sqrt{\pi} S\left(\frac{x}{\sqrt{\pi}}\right) \cos\left(\frac{x^2}{2}\right) + \cos\left(\frac{x^2}{2}\right)$$

where

$C(x)$ is the Fresnel C integral

$S(x)$ is the Fresnel S integral