$$\frac{x^3}{e^x-1}=\frac{e^{-x}x^3}{1-e^{-x}}=e^{-x}x^3\sum_{k=0}^{\infty}e^{-kx}=\sum_{k=1}^{\infty}x^3e^{-kx}$$

$$\int_0^{\infty}x^3e^{-kx}dx=-\frac{1}{k}\int_0^{\infty}x^3de^{-kx}=-\frac{1}{k}\left(-3\int_0^{\infty}x^2e^{-kx}dx\right)=$$

$$=-\frac{3}{k^2}\int_0^{\infty}x^2de^{-kx}=-\frac{3}{k^2}\left(-2\int_0^{\infty}xe^{-kx}dx\right)=$$

$$=-\frac{6}{k^3}\int_0^{\infty}xde^{-kx}=\frac{6}{k^3}\int_0^{\infty}e^{-kx}dx=\frac{6}{k^4}$$

$$\int_0^{\infty}\frac{x^3dx}{e^x-1}=\sum_{k=1}^{\infty}\int_0^{\infty}x^3e^{-kx}dx=6\sum_{k=1}^{\infty}\frac{1}{k^4}=6\zeta(4)=6\frac{\pi^4}{90}=\frac{\pi^4}{15}$$