Task 1. *Determine if the following series converges:* $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$.

Solution. Let a_n be the nth term of the series, i.e. $a_n = \frac{(n!)^2}{(2n)!}$. Use d'Alembert's ratio test. We have:

$$\frac{a_{n+1}}{a_n} = \frac{((n+1)!)^2}{(2(n+1))!} \cdot \frac{(2n)!}{(n!)^2} = \frac{(n+1)^2}{(2n+1)(2n+2)} \to \frac{1}{2 \cdot 2} = \frac{1}{4}$$

as $n \to \infty$. Since $\frac{1}{4} < 1$, we conclude that the series converges.

Answer: this series converges. $\qquad\square$

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