

Find a and b such that $(x+2)$ and $(x-2)$ are factors of $p(x)=ax^4+2x^3-3x^2+bx-4$

Let a_1, a_2, a_3 and a_4 be roots (may be complex) of $p(x)$.

$(x+2)$ and $(x-2)$ are factors of $p(x) \Rightarrow$ we can assume $a_3 = 2, a_4 = -2$.

Factor $p(x)$:

$$\begin{aligned} P(x) &= a(x - a_1)(x - a_2)(x - a_3)(x - a_4) = a(x - a_1)(x - a_2)(x - 2)(x + 2) = \\ &= a(x^2 - (a_1 + a_2)x + a_1 a_2)(x^2 - 4) = a(x^4 - (a_1 + a_2)x^3 + (a_1 a_2 - 4)x^2 + 4(a_1 + a_2)x - 4a_1 a_2) = \\ &= ax^4 - a(a_1 + a_2)x^3 + a(a_1 a_2 - 4)x^2 + 4a(a_1 + a_2)x - 4a a_1 a_2 = ax^4 + 2x^3 - 3x^2 + bx - 4 \end{aligned}$$

Coefficients beside the same powers of x must be equal:

$$a = a \quad (1)$$

$$-a(a_1 + a_2) = 2 \quad (2)$$

$$a(a_1 a_2 - 4) = -3 \quad (3)$$

$$4a(a_1 + a_2) = b \quad (4)$$

$$-4a a_1 a_2 = -4 \quad (5)$$

$$(2) \Rightarrow 4a(a_1 + a_2) = 2 \cdot (-4) = -8$$

$$(4) \Rightarrow 4a(a_1 + a_2) = b = -8$$

$$(5) \Rightarrow a_1 a_2 = 1/a$$

$$(3) \Rightarrow a(1/a - 4) = -3 \Rightarrow -4a = -3 \Rightarrow a = \frac{3}{4}$$

So, we get $a = \frac{3}{4}, b = -8$