Without actual division prove that $2x^4 - 5x^3 + 5x - 2$ are divisible by $x^2 - 3x + 2$.

Solution: Solve the equation $x^2 - 3x + 2 = 0$, we have roots $x_1 = 1, x_2 = 2$.

Let
$$f(x) = 2x^4 - 5x^3 + 5x - 2$$
, then $f(x_1) = f(1) = 2 - 5 + 5 - 2 = 0$, $f(x_2) = f(2) = 2 \cdot 16 - 5 \cdot 8 + 5 \cdot 2 - 2 = 0$.

So as soon as $f(x_1) = 0$ and $f(x_2) = 0$ that $2x^4 - 5x^3 + 5x - 2$ are divisible by $x^2 - 3x + 2$.