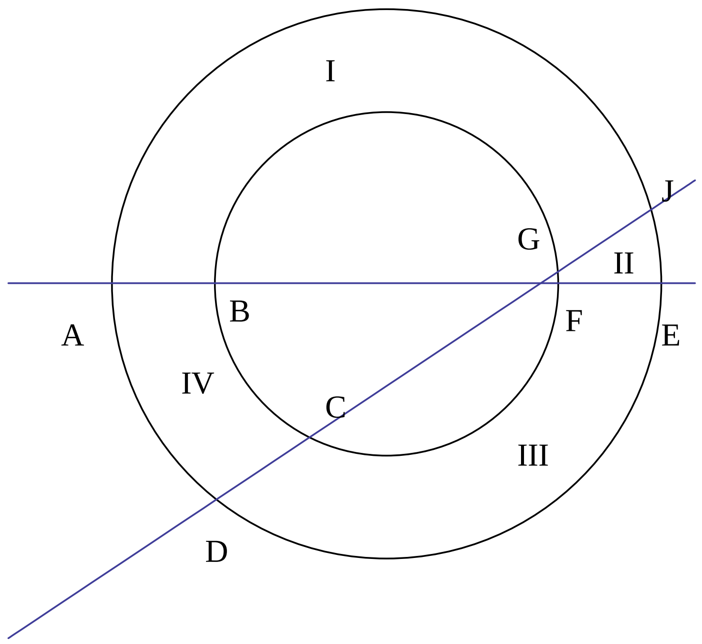
Lines y=2x/3 ; y=5 cut the ring formed by circles (x-3)^2+(y-5)^2=64 and (x-3)^2+(y-5)^2=25 into four parts. Find the area of each of the four parts.



We have

$$S_I + S_{II} = S_{III} + S_{IV} = \pi\left(8^2 - 5^2\right)\Big/2 = \frac{39\pi}{2}$$

Let’s find points of intersection

A(-5,5)

E(11,5)

B(-2,5)

F(8,5)

D$\left(\frac{3}{13}\left(19-\sqrt{751}\right), \frac{2}{13}\left(19-\sqrt{751}\right)\right)$

J$\left(\frac{1}{2}\left(\frac{114}{13}+\frac{6\sqrt{751}}{13}\right), \frac{2}{13}\left(19+\sqrt{751}\right)\right)$

C$\left(\frac{3}{13}\left(19-2\sqrt{61}\right), \frac{2}{13}\left(19-2\sqrt{61}\right)\right)$

G$\left(\frac{1}{2}\left(\frac{114}{13}+\frac{12\sqrt{61}}{13}\right), \frac{2}{13}\left(19+2\sqrt{61}\right)\right)$

$$y-5 = \pm\sqrt{64-(x-3)^2}$$

$$y-5 = \pm\sqrt{25-(x-3)^2}$$

$$S_{IV} = \int_{-5}^{\frac{3}{13}\left(19-2\sqrt{61}\right)} \sqrt{64-(x-3)^2}\,dx - \int_{-2}^{\frac{3}{13}\left(19-2\sqrt{61}\right)} \left(5-\left(-\sqrt{25-(x-3)^2}\right)-5\right)dx -$$

$$-\int_{\frac{3}{13}\left(19-\sqrt{751}\right)}^{\frac{3}{13}\left(19-2\sqrt{61}\right)} \left(\frac{2x}{3}+\sqrt{64-(x-3)^2}-5\right)dx \approx 19.1654$$

Then

$$S_{III} \approx 42.0956$$

Similar

$$S_{II} = \int_{\frac{1}{2}\left(\frac{114}{13}+\frac{12\sqrt{61}}{13}\right)}^{11} \sqrt{64-(x-3)^2}\,dx - \int_{\frac{1}{2}\left(\frac{114}{13}+\frac{12\sqrt{61}}{13}\right)}^{8} \left(\sqrt{25-(x-3)^2}+5-5\right)dx -$$

$$-\int_{\frac{1}{2}\left(\frac{114}{13}+\frac{12\sqrt{61}}{13}\right)}^{\frac{1}{2}\left(\frac{114}{13}+\frac{6\sqrt{751}}{13}\right)} \left(\sqrt{64-(x-3)^2}+5-\frac{2x}{3}\right)dx \approx 3.76674$$

$$S_I = 57.4943$$