

$$z = 3x_1 + 3x_2 + 13x_3 \rightarrow \max$$

$$-3x_1 + 6x_2 + 7x_3 \leq 8$$

$$6x_1 - 3x_2 + 7x_3 \leq 8$$

$$x_i \in \{0, 1, 2, 3, 4, 5\}$$

$$\begin{cases} -3x_1 + 6x_2 + 7x_3 \leq 8 \\ 6x_1 - 3x_2 + 7x_3 \leq 8 \end{cases} \text{ "-" } \Rightarrow 9x_1 - 9x_2 \leq 0 \Rightarrow 0 \leq x_1 \leq x_2$$

$$\text{"+" : } 3x_1 + 3x_2 + 14x_3 \leq 16 \Rightarrow 0 \leq 3x_1 + 3x_2 \leq 16 - 14x_3 \Rightarrow 14x_3 \leq 16 \Rightarrow 0 \leq x_3 \in \mathbb{Z} \Rightarrow x_3 \in \{0, 1\}$$

$$a) x_3 = 0$$

$$\begin{cases} z = 3x_1 + 3x_2 \rightarrow \max \\ -3x_1 + 6x_2 \leq 8 \\ 6x_1 - 3x_2 \leq 8 \\ x_i \in \{0, 1, 2, 3, 4, 5\} \end{cases}$$

$$0 \leq 3x_1 + 3x_2 \leq 16, 0 \leq x_1 \leq x_2 \leq 5$$

$$\max : x_1 + x_2 = 5$$

$$x_1 = 0, x_2 = 5, x_3 = 0 \Rightarrow z = 15$$

$$x_1 = 1, x_2 = 4, x_3 = 0 \Rightarrow z = 15$$

$$x_1 = 2, x_2 = 3, x_3 = 0 \Rightarrow z = 15$$

$$b) x_3 = 1$$

$$\begin{cases} z = 3x_1 + 3x_2 + 13 \rightarrow \max \\ -3x_1 + 6x_2 \leq 1 \\ 6x_1 - 3x_2 \leq 1 \\ x_i \in \{0, 1, 2, 3, 4, 5\} \end{cases}$$

$$0 \leq 3x_1 + 3x_2 \leq 2, 0 \leq x_1 \leq x_2 \leq 5$$

$$x_1 + x_2 = 0 \Rightarrow x_1 = x_2 = 0, x_3 = 1 \Rightarrow z = 13$$

So, solutions are :

$$(0, 5, 0), (1, 4, 0), (2, 3, 0), z_{\max} = 15$$