Question #81299

A Four cylinder vertical engine has cranks 300 cm long. The planes of rotation of the first, third and fourth cranks are 750 mm, 1050 mm and 1650 mm respectively from that of the second crank and their reciprocating masses are 150 kg, 400 kg and 250 kg respectively. Find the mass of the reciprocating parts for the second cylinder and relative angular position of the cranks in order that the engine may be in complete primary balance.

Answer:

The primary force, applied to the cranks *i*, is given by

$$F_{pi} = M_i \omega^2 R_i, \tag{1}$$

where M_i is the mass of the piston *i*, R_i is the length of the crank, ω is the angular velocity of the shaft.

For the dynamic balance the sum of forces and moments about any plane should be zero. Let us consider the forces and moments about plane of crank 2. Taking into account, that the cranck length and rotational velocity are the same for each crank, we can write down the following system of equations (assume the position of the crank 1 is 0 degrees):

$$\sum F_x = 0:\ \ 150 + m_x + 400\cos\theta_3 + 250\cos\theta_4 = 0, \tag{2}$$

$$\sum F_y = 0:\ m_y + 400\sin\theta_3 + 250\sin\theta_4 = 0, \tag{3}$$

$$\sum M_{2x} = 0:\ -0.75\cdot 150 + 1.05\cdot 400\cos\theta_3 + 1.65\cdot 250\cos\theta_4 = 0, \tag{4}$$

$$\sum M_{2x} = 0:\ \ 1.05\cdot 400\sin\theta_3 + 1.65\cdot 250\sin\theta_4 = 0, \tag{5}$$

where θ_3 and θ_4 are the positions of the cranks 3 and 4, respectively,

m_x and m_y are the components of the reciprocating mass 2.

From (4) and (5), we can define the following relations:

$$\cos\theta_4 = \frac{112.5 - 420\cos\theta_3}{412.5}, \tag{6}$$

$$\sin\theta_4 = -\frac{420\sin\theta_3}{412.5}. \tag{7}$$

Let us use the Pythagorean identity:

$$\cos^2\theta_4 + \sin^2\theta_4 = 1,$$

$$\left(\frac{112.5 - 420\cos\theta_3}{412.5}\right)^2 + \left(\frac{420\sin\theta_3}{412.5}\right)^2 = 1,$$

$$112.5^2 - 2\cdot 112.5\cdot 420\cos\theta_3 + 420^2(\cos^2\theta_3 + \sin^2\theta_3) = 412.5^2,$$

$$\cos\theta_3 = \frac{112.5^2 + 420^2 - 412.5^2}{2\cdot 112.5\cdot 420} = 0.2.$$

Substitute into (6):

$$\cos\theta_4 = \frac{112.5 - 420 \cdot 0.2}{412.5} = 0.0691.$$

Taking into account (5), we see, that θ_3 and θ_4 have opposite signs. Thus, the positions of the masses B and C are

$$\theta_3 = 78.5° \quad \text{and} \quad \theta_4 = -86.0°\,.$$

Substitute into (2) and (3):

$$150 + m_x + 400 \cdot 0.2 + 250 \cdot 0.0691 = 0,$$

$$m_x = -150 - 400 \cdot 0.2 - 250 \cdot 0.0691 = -247.3,$$

$$m_y + 400\sin 78.5° + 250\sin(-86.0°) = 0,$$

$$m_y = -142.5.$$

Thus the reciprocating mass 2 and its position are:

$$m = \sqrt{m_x^2 + m_y^2} = \sqrt{247.3^2 + 142.5^2} = 285.4 \text{ kg},$$

$$\tan\theta_2 = \frac{m_y}{m_x},$$

$$\theta_2 = \operatorname{atan}\frac{142.5}{247.3} + 180° = 210.0°.$$