

Question #81296

The crank and connecting rod of a steam engine are 0.3 m and 1.5 m in length. The crank rotates at 180 rpm. Clock wise. Determine the velocity and acceleration of the piston when the crank is at 40 degrees from the inner dead centre position. Also determine the position of the crank for zero acceleration of the piston.

Answer:

Figure 1 shows the position of the mechanism at the given moment.

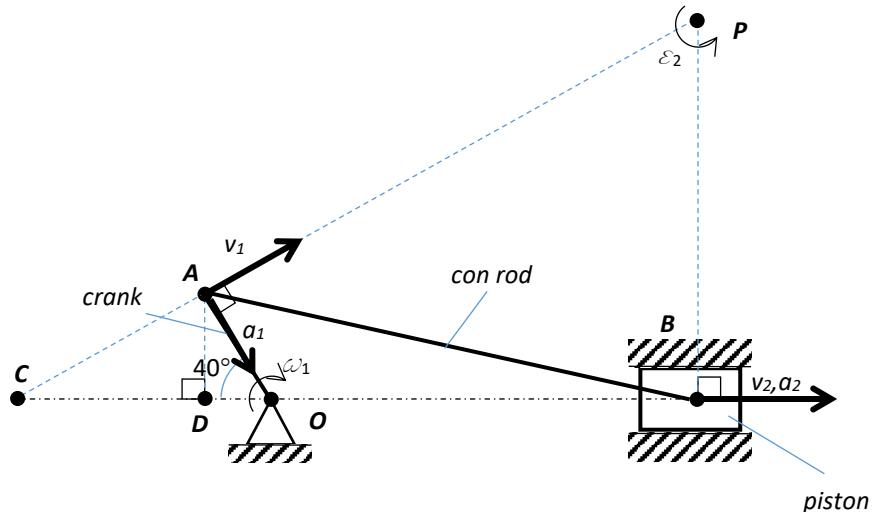


FIGURE 1. Crank-piston mechanism

The rotational velocity of the crank is

$$\omega_1 = \frac{180\pi}{30} = 18.85 \text{ rad/s.}$$

The velocity v_A of the point A is directed perpendicularly to the crank and has magnitude of

$$v_A = OA \cdot \omega_1 = 0.3 \cdot 18.5 = 5.65 \text{ m/s.}$$

The velocity v_B of the piston B equals to the horizontal component of v_A

$$v_B = v_A \cdot \sin 40^\circ = 5.65 \cdot 0.6428 = 3.63 \text{ m/s.}$$

The acceleration a_1 of the point A is directed along the crank to O and has magnitude of

$$a_1 = OA \cdot \omega_1^2 = 0.3 \cdot 18.5^2 = 106.6 \text{ m/s}^2.$$

In triangle ADC we have

$$AD = OA \cdot \sin 40^\circ = 0.3 \cdot 0.6428 = 0.193 \text{ m,}$$

$$AC = OA \cdot \tan 40^\circ = 0.3 \cdot 0.839 = 0.252 \text{ m,}$$

$$CD = AC \cdot \sin 40^\circ = 0.252 \cdot 0.6428 = 0.162 \text{ m.}$$

The distances BD and BC equal to

$$BD = \sqrt{AB^2 - AD^2} = \sqrt{1.5^2 - 0.193^2} = 1.49 \text{ m},$$

$$BC = BD + CD = 1.49 + 0.162 = 1.65 \text{ m}.$$

Now, we can find the distances AP and BP to the current centre P of the rotation of the conrod AB as follow

$$\frac{AC}{CP} = \frac{CD}{BC},$$

$$CP = AC \cdot \frac{BC}{CD} = 0.252 \cdot \frac{1.65}{0.162} = 2.57 \text{ m},$$

$$AP = CP - AC = 2.57 - 0.252 = 2.31 \text{ m},$$

$$\frac{AD}{BP} = \frac{CD}{BC},$$

$$BP = AD \cdot \frac{BC}{CD} = 0.193 \cdot \frac{1.65}{0.162} = 1.97 \text{ m}.$$

Thus, the current rotational acceleration ε_2 of the conrod is

$$\varepsilon_2 = \frac{a_1}{AP} = \frac{106.6}{2.31} = 46.1 \text{ rad/s}^2.$$

Finally, the acceleration of the piston is

$$a_2 = BP \cdot \varepsilon_2 = 1.97 \cdot 46.1 = 90.5 \text{ m/s}^2.$$

The piston has zero acceleration at the dead points. The positions of the crank at the piston dead points are 0 and 180 degrees.