Question

A cooper tube 55 mm in outside diameter and of wall thickness 5 mm fits loosely over a solid steel circular bar 40 mm in diameter. The two members are fastened together by two metal pins each 8 mm in diameter and passing transversely through both members, one pin being near each and of the assembly. At room temperature the assembly is just stress free when the pins are in position. The temperature of the entire assembly is then raised 40°C. Calculate the average shear stress in the pins. For cooper E=90GPa, a=18x10¯6/°C; for steel E=200GPa, a=12x10¯6/°C.

Solution

Since the pins are transverse, the radial expansion of the tubes assembly may be ignored in this case.

As the temperature increases, the assembly will tend to expand. However, steel shaft will experience higher thermal expansion force due to its higher young's modulus. As a result, both will be under thermal expansion. The difference between the thermal expansion forces will be carried by the pins and thus causing shear stress.

Therefore;

$$F = \alpha A E \Delta T$$

Where

F = Thermal expansion load

α = Thermal expansion coefficient

A = Cross-sectional area

E = Young's modulus

ΔT = Change in temperature = 40 – 20 (room temperature) = 20°C

For steel shaft;

$$F_s = 12 * 10^{-6} * \pi * \frac{0.04^2}{4} * 200 * 10^9 * 20 = 60318.58\ N$$

For copper tube

$$F_c = 18 * 10^{-6} * \pi * \frac{0.055^2 - 0.045^2}{4} * 90 * 10^9 * 20 = 25446.90\ N$$

Then;

$$F_s - F_c = 60318.58 - 25446.90 = 34871.68\ N$$

This is the load that will be carried by the two steel pins in order to keep the assembly intact.

In this regard, the load carried by each pin is;

$$F_{pin} = \frac{34871.68}{2} = 17435.84\ N$$

And then, shear stress will be;

$$\tau = \frac{Load}{Area} = \frac{17435.84}{\frac{\pi d^2}{4}} = \frac{17435.84}{\frac{\pi * 0.008^2}{4}} = 346875004.9\ Nm^{-2} \approx \mathbf{346.88\ MN.m^{-2}}$$

Answer

The average shear stress in the pins will be;

$$\tau = 346.88\ MN.m^{-2}$$