Question #79126

0.5kg of air is compressed reversibly and adiabatically from 80kPa, 60°C to 0.4Mpa and is then expanded at constant pressure to the original volume. Calculate the heat transfer and work transfer for the whole path.

Take R = 0.247kJ/kg.K, Cp = 1.005kJ/kg.K, Cv = 0.718kJ/kg.K

Answer:

Assume the air is an ideal gas. Thus, from the ideal gas equation we have:

$$pV = mRT, \tag{1}$$

where p, V and T are pressure, volume and absolute temperature, respectively,

m = 0.5 kg – the mass of the air (which remains constant),

R = 0.247 kJ/kg.K – the gas constant of air.

From (1) we can find the initial volume of the processed air (T_0 = 60°C + 273K = 333K):

$$V_0 = \frac{mRT_0}{p_0} = \frac{0.5 \cdot 0.247 \cdot 333}{80} = 0.514 \text{ m}^3.$$

Since the compression is reversible and adiabatic, the heat transfer is zero (Q_c = 0), and the transferred work is given by:

$$W_c = -\alpha m R T_0 \left(\left(\frac{p_1}{p_0} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right), \tag{2}$$

where

$$\gamma = \frac{C_p}{C_V}, \tag{3}$$

the adiabatic index,

α = 2.5 – the number of degrees of freedom (which is 5 for the air) divided by two.

Substitute into (3) and (2):

$$\gamma = \frac{1.005}{0.718} = 1.400,$$

$$W_c = -2.5 \cdot 0.5 \cdot 0.247 \cdot 333 \left(\left(\frac{400}{80} \right)^{\frac{1.4-1}{1.4}} - 1 \right) = -60.00 \text{ kJ}.$$

The final volume of the air could be determined from the equation of an adiabatic process:

$$p_0 V_0^\gamma = p_1 V_1^\gamma, \tag{4}$$

$$V_1 = V_0 \left(\frac{p_0}{p_1} \right)^{\frac{1}{\gamma}} = 0.514 \cdot \left(\frac{80}{400} \right)^{\frac{1}{1.4}} = 0.163 \text{ m}^3.$$

The temperature at the end of compression could be defined from (1) as follow:

$$T_1 = \frac{p_1 V_1}{mR} = \frac{400 \cdot 0.163}{0.5 \cdot 0.247} = 527.3\ \mathrm{K}.$$

For the second process, which is isobaric expansion, the heat and the work transfer, respectively, are given by:

$$Q = mC_p \Delta T, \tag{5}$$

$$W = p\Delta V, \tag{6}$$

where ΔT and ΔV are change in temperature and volume, respectively.

From (1), the final temperature of the air is:

$$T_2 = \frac{p_2 V_2}{mR} = \frac{400 \cdot 0.514}{0.5 \cdot 0.247} = 1664.8\ \mathrm{K}.$$

Substitute into (5) and (6):

$$Q_e = 0.5 \cdot 1.005 \cdot (1664.8 - 527.3) = 571.7\ \mathrm{kJ},$$

$$W_e = 400 \cdot (0.514 - 0.163) = 140.5\ \mathrm{kJ}.$$

The total heat transfer is:

$$Q = Q_c + Q_e = 571.7\ \mathrm{kJ}.$$

The total work transfer is:

$$W = -60 + 140.5 = 80.5\ \mathrm{kJ}.$$