Question # 78928

At 15°C and 1.05bar occupies a volume of 0.02m³. The air is heated at constant volume until the pressure is 4.2bar and then cooled at constant pressure back to original temperature. Calculate the net heat follow to and from the air and net entropy change

Answer:

Assume the air is an ideal gas. Thus, from the ideal gas equation we have:

$$pV = mRT, \tag{1}$$

where *p*, *V* and *T* are pressure, volume and absolute temperature respectively,

m is the mass of air (which remains constant),

R = 287 $J.kg^{-1}K^{-1}$ – the gas constant of air.

From (1) we can find the mass of the processed air at the initial conditions (T_1 = 15°C + 273K = = 288K):

$$m = \frac{pV}{RT} = \frac{1.05 \cdot 10^5 \cdot 0.02}{287 \cdot 288} = 0.0254 \text{ kg}.$$

Since the heating occurs at a constant volume, i.e. *mR*/*V* = *const*, (1) yields:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}, \tag{2}$$

From (2) we can find the temperature of the air after heating:

$$T_2 = T_1 \frac{p_2}{p_1} = 288 \cdot \frac{4.2}{1.05} = 1152 \text{ K}.$$

The net heat flow *Q* is given by:

$$Q = mC\Delta T, \tag{3}$$

where *C* is a specific heat capacity of the air in the process (for an ideal air at *V* = *const* $C_v = \frac{5}{2}R$, at *p* = *const* $C_p = \frac{7}{2}R$),

ΔT is the change in temperature of the air during a process.

Substitution into (3) gives us the net heat flow during the heating (at *V* = *const*):

$$Q_H = 0.0254 \cdot \frac{5}{2} \cdot 287 \cdot (1152 - 288) = 15{,}750 \text{ J},$$

and during the cooling (at *p* = *const*):

$$Q_C = 0.0254 \cdot \frac{7}{2} \cdot 287 \cdot (288 - 1152) = -22{,}050 \text{ J}.$$

The net entropy change ΔS is given by:

$$Q = mC \ln\left(\frac{T_f}{T_i}\right), \tag{4}$$

where T_i and T_f are the initial and the final temperatures of the air in a process.

Substitution into (4) gives us the net entropy change during the heating (at V = *const*):

$$\Delta S_H = 0.0254 \cdot \frac{5}{2} \cdot 287 \cdot \ln\left(\frac{1152}{288}\right) = 25.27\ \mathrm{J.K^{-1}},$$

and during the cooling (at p = *const*):

$$\Delta S_C = 0.0254 \cdot \frac{7}{2} \cdot 287 \cdot \ln\left(\frac{288}{1152}\right) = -35.38\ \mathrm{J.K^{-1}}.$$