Answer on Question #64289-Engineering-Civil and Environmental Engineering

A workshop table has an equilateral triangular top, each side of which is 900mm, the legs being at the three corners. A load of 500 N is placed on the table at a point distant 325 mm from one leg and 625 mm from another. What is the load in each of the three legs?

Solution

1. Sit the triangle on the x-axis, with the left vertex A at origin, so that $(0,900)$ is the other vertex B.

The third vertex C has coordinates $(450, 450\sqrt{3})$.

2. The loading point D forms another triangle with a common base of the first, assuming point D is within the triangle with sides 900, 325 and 625.

3. Assume $mAD = 325$, hence $mBD = 625$.

4. Drop a perpendicular from D to AB, meeting AB at E.

Denote height mDE as h, and mAE as x, then $mEB = 900 - x$.

5. Using Pythagoras Theorem, we have two equations:

$$h^2 + x^2 = 325^2 \qquad (1)$$

$$h^2 + (900 - x)^2 = 625^2 \qquad (2)$$

6. Rewrite (1) as $h^2 = 325^2 - x^2$ and substitute in (2). Solve for x, and hence h.

$$325^2 - x^2 + (900 - x)^2 = 625^2 \rightarrow x = \frac{875}{3}$$

$$h^2 = 325^2 - \left(\frac{875}{3}\right)^2$$

$$h = 50\frac{\sqrt{74}}{3}$$

7. Denote leg reactions as $R_a, R_b \; and \; R_c$ respectively, and the load P=500N.

8. Take moments about x-axis.

$$Ph - R_c(450\sqrt{3}) = 0$$

$$\boldsymbol{R_c = \frac{500\sqrt{74}}{3^{\frac{7}{2}}} = 92\ N.}$$

9. Take moments about the y-axis.

$$Px - R_b 900 - R_c 450 = 0$$

$$\boldsymbol{R_b = \frac{4375 \cdot 3^{\frac{3}{2}} - 750\sqrt{(74)}}{3^{\frac{9}{2}}} = 116\ N.}$$

10. Finally,

$$R_a = P - R_b - R_c = 500 - \frac{500\sqrt{74}}{3^{\frac{7}{2}}} - \frac{4375 \cdot 3^{\frac{3}{2}} - 750\sqrt{74}}{3^{\frac{9}{2}}} = 292\ N.$$