Answer on Question #49308, Engineering, Other

A plane diffraction grating is illuminated with a monochromatic light of wavelength 5100 Å. If the angle of diffraction for third order maxima is 22.5°, calculate the number of lines in one inch of the grating surface. Also, if this grating is illuminated with another light of wavelength 5200 Å, then find the angle of separation of 5100 Å and 5200 Å lines in second order spectrum.

Solution:

When light is normally incident on the grating, the diffracted light will have maxima at angles θ_m given by:

$$d \sin \theta_m = m\lambda$$

Thus, the slit separation is

$$d = \frac{m\lambda}{\sin \theta_m} = \frac{3 * 5100 * 10^{-10}}{\sin 22.5°} = 4 * 10^{-6} \text{ m}$$

The number of lines per meter is

$$N = \frac{1}{d} = \frac{1}{4 * 10^{-6}} = 250000 \; lines/m$$

The number of lines per inch is

$$N * 0.0254 = 250000 * 0.0254 = 6350 \; lines/inch$$

The angle of separation of 5100 Å and 5200 Å lines in second order spectrum is

$$\Delta\theta = \theta_2 - \theta_1$$

where

$$\theta_2 = \sin^{-1}\left(\frac{2\lambda_2}{d}\right) = \sin^{-1}\left(\frac{2 * 5200 * 10^{-10}}{4.* 10^{-6}}\right) = 15.07°$$

$$\theta_1 = \sin^{-1}\left(\frac{2\lambda_1}{d}\right) = \sin^{-1}\left(\frac{2 * 5100 * 10^{-10}}{4.* 10^{-6}}\right) = 14.77°$$

$$\Delta\theta = 15.07° - 14.77° = 0.3°$$

Answer: $6350 \; lines/inch$; 0.3°.