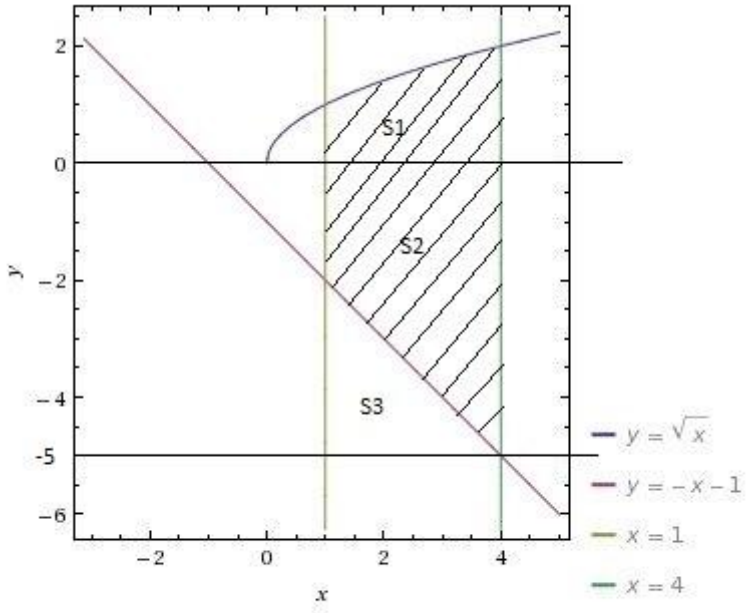
Question #47818, Engineering, Other

Problem

Find the area of the region bounded by the graphs of $y=\sqrt{x}$ and $y=-x-1$ between $x=1$ and $x=4$.

Solve

Plot of solution set:



We need to find a shaded area. For this we use the integral.

Let $y_1=\sqrt{x}$ and $y_2=-x-1$, $x_1=1$, $x_2=4$

$$S=S1+S2$$

$$S1=\int_{x_1}^{x_2}\sqrt{x}dx=\int_{x_1}^{x_2}x^{\frac{1}{2}}dx=x^{\frac{1}{2}+1}\cdot\frac{1}{\frac{1}{2}+1}\Bigg|_1^4=x^{3/2}\cdot\frac{2}{3}\Big|_1^4=4^{\frac{3}{2}}\cdot\frac{2}{3}-1^{\frac{3}{2}}\cdot\frac{2}{3}=\frac{14}{3}$$

$$S2+S3=(x_2-x_1)(0-(-5))=15 \qquad S3=\frac{1}{2}(x_2-x_1)(-2-(-5))=\frac{9}{2}$$

$$S2=15-S3=15-\frac{9}{2}=\frac{21}{2} \qquad S=S1+S2=\frac{14}{3}+\frac{21}{2}=\frac{28+63}{6}=\frac{91}{6}$$

Answer: area is $\frac{91}{6}$.