Answer on Question #46381, Engineering, Other

Task: Show that the curvilinear coordinate system defined by the following equations is orthogonal:

$x = uv\cos\varphi;$

$y = uv\sin\varphi;$

$z = \frac{(u^2 - v^2)}{2}$

Solution:

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} uv\cos\varphi \\ uv\sin\varphi \\ \frac{(u^2 - v^2)}{2} \end{pmatrix}$$

Derivatives of the radius vector:

$$\vec{r}_u = \begin{pmatrix} x_u \\ y_u \\ z_u \end{pmatrix} = \begin{pmatrix} v\cos\varphi \\ v\sin\varphi \\ u \end{pmatrix}; \vec{r}_v = \begin{pmatrix} x_v \\ y_v \\ z_v \end{pmatrix} = \begin{pmatrix} u\cos\varphi \\ u\sin\varphi \\ -v \end{pmatrix}; \vec{r}_\varphi = \begin{pmatrix} x_\varphi \\ y_\varphi \\ z_\varphi \end{pmatrix} = \begin{pmatrix} -uv\sin\varphi \\ vu\cos\varphi \\ 0 \end{pmatrix};$$

Scalar products:

$\vec{r}_u \cdot \vec{r}_v = uv\cos^2\varphi + uv\sin^2\varphi - uv = 0$

$\vec{r}_u \cdot \vec{r}_\varphi = -uv^2\cos\varphi\sin\varphi + uv^2\sin\varphi\cos\varphi = 0$

$\vec{r}_v \cdot \vec{r}_\varphi = -u^2v\cos\varphi\sin\varphi + u^2v\sin\varphi\cos\varphi = 0$

It means that $\vec{r}_u, \vec{r}_v, r_\varphi$ can be chosen as a basis and these vectors are orthogonal.