Answer on Question #46097-Engineering-Other

(a) Find the equation of the line which passes through $(1, \sqrt{3})$ and makes an angle 30^0 with the line $x - \sqrt{3}y - \sqrt{3} = 0$.

(b) Find the distance of the line obtained in part (a), from the origin by expressing it in the normal form. Also find the intercepts made by this line on the coordinate axes.

(c) Obtain the equation of the plane Q passing through the line L: $\frac{x-2}{2} = -\frac{y+1}{1} = \frac{z-3}{4}$ and which is perpendicular to the plane $P: x + 2y + z = 4$.

(d) Find the vertices, eccentricity, foci and asymptotes of the hyperbola. Also trace it. Under what conditions on (λ) the line $x - \lambda y + 2 = 0$ will be tangent to this hyperbola?

Solution

(a) The line $x - \sqrt{3}y - \sqrt{3} = 0$ $(y = \frac{1}{\sqrt{3}}x + 1)$ has slope $\frac{1}{\sqrt{3}} = \tan 30°$, so the slope of the line which makes an angle $30°$ with this line is equal to $\tan 0° = 0$ or $\tan 60° = \sqrt{3}$. Then the new line has equation $y = \sqrt{3}$ or $y = \sqrt{3}x$ (as they both pass through point $(1, \sqrt{3})$).

(b) The normal equations of lines $y = \sqrt{3}$ and $y = \sqrt{3}x$ are $y - \sqrt{3} = 0$ and $\frac{1}{2}y - \frac{\sqrt{3}}{2}x = 0$. Therefore the distances to the origin are equal to $\sqrt{3}$ and 0 respectively. The line $y = \sqrt{3}x$ passes through the origin.

The line $y = \sqrt{3}$ intersects Oy at $(0, \sqrt{3})$ and doesn't intersect Ox.

The line $y = \sqrt{3}x$ intersects Ox and Oy at (0,0).

(c) We have that

the line L passes through a point $A(2, -1, 3)$ in the direction of the vector $l(2, -1, 4)$,

the normal vector of the plane P has coordinates $p(1, 2, 1)$.

Let $n(a, b, c)$ be normal vector of the plane Q passing through the line L and perpendicular to Q. Then Q passes through point A, whence its equation has the following form:

$$a(x - 2) + b(y + 1) + c(z - 3) = 0.$$

Notice that n must be perpendicular to both vectors $l(2, -1, 4)$ and $p(1, 2, 1)$, and therefore we can choose n to be the cross product of these vectors:

$$n = l \times p.$$

Thus

$$n = l \times p = (2, -1, 4) \times (1, 2, 1) = \left(\begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix}, \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} \right) =$$

$$= (-1 \cdot 1 - 2 \cdot 4, 4 \cdot 1 - 1 \cdot 2, 2 \cdot 2 - 1 \cdot (-1)) = (-9, 2, 5).$$

Hence Q has the following equation:

$$-9(x - 2) + 2(y + 1) + 5(z - 3) = 0$$

$$-9x + 18 + 2y + 2 + 5z - 15 = 0$$

$$-9x + 2y + 5z + 5 = 0.$$

(d) For the hyperbola with equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

We have:

a) the vertices in points $(-a, 0)$ and $(a, 0)$

b) eccentricity ε is equal $\varepsilon = \frac{c}{a}$ where $c = \sqrt{a^2 + b^2}$

c) the foci in points $(-c, 0)$ and $(c, 0)$, $c = \sqrt{a^2 + b^2}$

d) asymptotes of the hyperbola: $y = \frac{b}{a}x, y = -\frac{b}{a}x$

e) the line passing through the point of hyperbola (x_0, y_0) and which is tangent to the hyperbola has equation:

$$\frac{x \cdot x_0}{a^2} - \frac{y \cdot y_0}{b^2} = 1.$$

So we have hyperbola with equation:

$$\frac{x^2}{8} - \frac{y^2}{4} = 1.$$

Therefor hyperbola has:

a) the vertices in points $(-2\sqrt{2}, 0)$ and $(2\sqrt{2}, 0)$

b) $c = \sqrt{a^2 + b^2} = \sqrt{8 + 4} = 2\sqrt{3}$, so eccentricity ε is equal $\varepsilon = \frac{c}{a} = \frac{2\sqrt{3}}{2\sqrt{2}} = \sqrt{\frac{3}{2}}$

c) the foci in points $(-2\sqrt{3}, 0)$ and $(2\sqrt{3}, 0)$

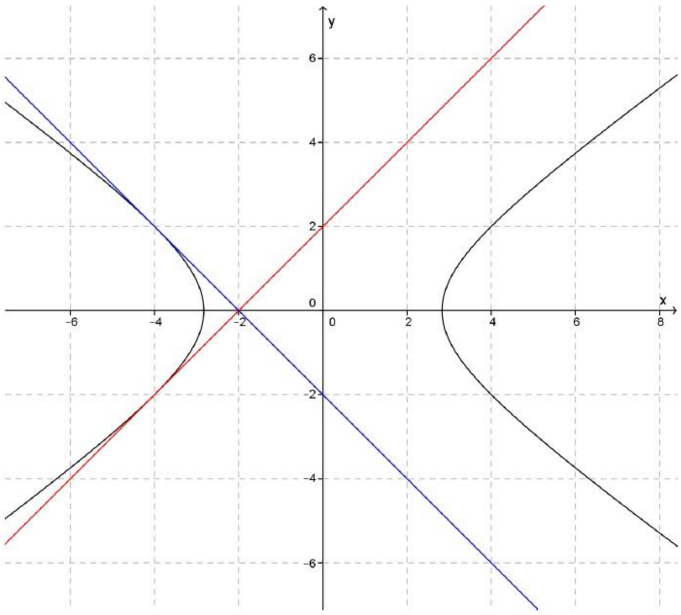
d) asymptotes of the hyperbola: $y = \frac{2}{2\sqrt{2}}x = \frac{\sqrt{2}}{2}x, y = -\frac{2}{2\sqrt{2}}x = -\frac{\sqrt{2}}{2}x$

e) the line passing through the point of hyperbola (x_0, y_0) and which is tangent to the hyperbola has equation:

$$\frac{x \cdot x_0}{8} - \frac{y \cdot y_0}{4} = 1.$$

We should find such λ that $-\frac{x}{2} + \frac{\lambda y}{2} = 1$. Then $-\frac{1}{2} = \frac{x_0}{8}$ and $\frac{\lambda}{2} = \frac{y_0}{4}$ or $x_0 = -4 \; and \; y_0 = -2\lambda$, but $\frac{{x_0}^2}{8} - \frac{{y_0}^2}{4} = 1$ so $2 - \lambda^2 = 1$. Therefore $\lambda = \pm 1$.

Hence there are two tangent lines $x + y + 2 = 0$ (blue line) and $x - y + 2 = 0$ (red line). There are two tangent lines because the hyperbola is symmetric with respect to x-axis.



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