Answer on Question #45759, Engineering, Other

Task:

a) Which of the following functions are 1-1 and which are onto? Justify your answer.

i) f : R $\to$ R0 given by f (x) = x^2 where R^0 is the set fx 2 Rjx 0g.

ii) f : R $\to$ R given by f (x) = x^2+ x + 1.

b) Let a =(1+2p^2;p^3+2p^2;1+p^2)

and b =(1+p^2;0;1+p^2).

i) Find the direct cosines of a and b.

ii) Find the angle between a and b.

c) Check that the vectors $\bar{u}=(35;45;0)\bar{v}=(45;35;0)$

and $\bar{w}=(0;0;1)$ are orthonormal. Further, write the vector a = (1; 1; 2) as a linear combination of the vectors.

Solution:

a)

The function is injective or 1 to 1 if every element of the function's codomain is the image of at most one element of its domain.

The function f from a set X to a set Y is surjective (or onto), or a surjection, if every element y in Y has a corresponding element x in X such that f(x) = y.

i) f : R $\to$ R_0 given by f (x) = x^2

if R_0 =R\{0} then f is not onto because for y = −2 we can't find a x such that f (x) = x^2 and it is not 1 to 1, because f(1) = f(−1) = 1. If $R_0=[0;+\infty]$ then f is onto, because for every y in [0; +∞) exists x = $\sqrt{y}$ such that y=f(x) and it is not 1 to 1, because f(-1)=f(1).

ii) f : R $\to$ R given by f (x) = x^2+ x + 1.

This function is not onto, because for y = 0 we can't find x such that 0 = x^2+ x + 1 .And it is not 1 to 1 because f(1) = f(−2) = 3.

b) Let a =(1+2p2;p3+2p2;1+p2)

and b =(1+p2;0;1+p2).

i) Find the direct cosines of a and b.

ii) Find the angle between a and b.

$$\cos\angle(\bar{a},\bar{b})=\frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}=\frac{(1+p^2)(1+2p^2)+0(p^3+2p^2)+(1+p^2)^2}{\sqrt{(1+p^2)^2+(1+p^2)^2}\sqrt{(1+2p^2)^2+(p^3+2p^2)^2+(1+p^2)^2}}=$$

$$=\frac{3p^4+5p^2+2}{(1+p^2)\sqrt{2}\sqrt{p^6+4p^5+9p^4+6p^2+2}}\Rightarrow$$

$$\angle(\bar{a},\bar{b})=\arccos\left(\frac{3p^4+5p^2+2}{(1+p^2)\sqrt{2}\sqrt{p^6+4p^5+9p^4+6p^2+2}}\right)$$

c) Check that the vectors $\overline{u} = (35;45;0)\overline{v} = (45;35;0)$ and $\overline{w} = (0;0;1)$ are orthonormal. Further, write the vector a = (1; 1; 2) as a linear combination of the vectors.

$\overline{u} = (35;45;0)\overline{v} = (45;35;0)$ and $\overline{w} = (0;0;1)$ are orthonormal if $\overline{u} \cdot \overline{v} \cdot \overline{w} = 0$
so $\overline{u} \cdot \overline{v} \cdot \overline{w} = 35*45*0 + 45*35*0 + 0*0*1 = 0$ they are orthonormal.

Further, write the vector a = (1; 1; 2) as a linear combination of the vectors
$\overline{u} = (35;45;0)\overline{v} = (45;35;0)$ **and** $\overline{w} = (0;0;1)$

$$\overline{a} = x\overline{u} + y\overline{v} + z\overline{w}$$

$$(1;1;2) = x(35;45;0) + y(45;35;0) + z(0;0;1)$$

$$(1;1;2) = (35x + 45y;45x + 35y; z)$$

$$\begin{cases} 35x + 45y = 1 \\ 45x + 35y = 1 \\ z = 2 \end{cases}$$

$$\begin{cases} x = \dfrac{1}{80} \\ y = \dfrac{1}{80} \\ z = 2 \end{cases}$$

So linear combination of the vectors is (1/80;1/80;2)