Answer on Question #45529-Engineering-SolidWorks-CosmoWorks-Ansys

A Box contains 12 balls of which 3 are white and 9 are red. A sample of 3 balls is selected at random from the box. Find the moment generating function of X and hence find mean and standard deviation of the distribution.

Solution

Suppose that a box contains white balls (proportion equals $p = \frac{3}{12} = \frac{1}{4}$) and red balls $p = \frac{9}{12} = \frac{3}{4}$. A random sample of $n = 3$ balls is selected with replacement.

Then

$$P(X = x) = \frac{n!}{x!\,(n - x!)} p^x q^{n-x}.$$

Then the moment generating function is given by

$$M_x(t) = \sum_{x=0}^{n} e^{xt} \frac{n!}{x!\,(n - x!)} p^x q^{n-x} = \sum_{x=0}^{n} (pe^t) \frac{n!}{x!\,(n - x!)} q^{n-x} = (q + pe^t)^n.$$

In our case

$$M_x(t) = \left(\frac{3}{4} + \frac{1}{4} e^t\right)^3.$$

<u>The mean is</u>

$$E(x) = \frac{dM_x(t)}{dt}(t = 0\,) = 3\left(\frac{3}{4} + \frac{1}{4} e^t\right)^2 \frac{1}{4} e^t (t = 0\,) = \frac{3}{4}.$$

$$E(x^2) = \frac{d^2\, M_x(t)}{dt^2}(t = 0) = 3\left(\frac{3}{4} + \frac{1}{4} e^t\right)^1 \frac{1}{4} e^t \left(\frac{3}{4} + 3\frac{1}{4} e^t\right)(t = 0\,) = \frac{3}{4}\left(\frac{3}{4} + \frac{3}{4}\right) = \frac{18}{16}.$$

The variance

$$Var(x) = E(x^2) - \big(E(x)\big)^2 = \frac{18}{16} - \left(\frac{3}{4}\right)^2 = \frac{9}{16}.$$

<u>Standard deviation is</u>

$$\sigma = \sqrt{Var(x)} = \frac{3}{4}.$$