Answer on Question #45527, Engineering, Solid Works

In an examination taken by 500 candidates, the average and S.D of marks obtained are 40% and 10% respectively. Assuming normal distribution, find

(i) How many have scored above 60%;

(ii) How many will pass if 50% is fixed as the minimum marks for passing;

(iii) How many will pass if 40% is fixed as the minimum marks for passing;

(iv) What should be the minimum percentage of marks for passing so that 350 candidates pass.

Solution:

The normal distribution is the most widely known and used of all distributions. Because the normal distribution approximates many natural phenomena so well, it has developed into a standard of reference for many probability problems.

The parameters μ and σ are the mean and standard deviation, respectively, and define the normal distribution. The symbol e is the base of the natural logarithm and π is the constant pi.

The rule for a normal density function is

$$f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

There is a very strong connection between the size of a sample N and the extent to which a sampling distribution approaches the normal form. Many sampling distributions based on large N can be approximated by the normal distribution even though the population distribution itself is definitely not normal.

Based on the above information we can apply the following formula to calculate the find value. Note the given values N=500 (number of candidates), Mean=$\mu = 40$ and $\sigma = 10$.

We consider the first case. To find how many have scored above 60% we apply the following formula.

$$z = \frac{x - \mu}{\sigma}$$

We substitute the given values.

$$z = \frac{60 - 40}{10} = \frac{20}{10} = 2$$

We use the table which shows the area from 0 to Z. In our case $P(x \geq 60) = P(z \geq 2) = P(0 < z < \infty) - P(0 < z < 2)$ will be equal to 0.5 – (Area between 0 and 2) = $0.5 - 0{,}4772 = 0.0228$

According to the condition of the task we need to find the number of candidates scored above 60%.

The required number of candidates who scored more than 60% marks = $500 \cdot 0.0228 \approx 11$ (approximately).

Consider second case to find how many candidates will pass if 50% is fixed as the minimum marks for passing.

In given problem we apply the same formula to determinate the value of z.

$$\mathrm{z} = \frac{\mathrm{x} - \mu}{\sigma}$$

We substitute the given values.

$$z = \frac{50 - 40}{10} = \frac{10}{10} = 1$$

We use the table which shows the area from 0 to Z. In our problem $\mathrm{P}(\mathrm{x} \geq 50) = \mathrm{P}(\mathrm{z} \geq 1) = 0.5 - \mathrm{P}(0 < \mathrm{z} \leq 1)$ will be equal to 0.5 – (Area between 0 and 1) = $0.5 - 0{,}3413 = 0.1587$

Now we can find the number of candidates which will pass if 50% is fixed as the minimum marks for passing.

The required number of candidates is equal to

$500 \cdot 0.1587 = 79.35 \approx 79$ (approximately).

Next we have to find the how many candidates will pass if 40% is fixed as the minimum marks for passing.

As in previous part of task we apply the formula for determination z (the standard normal random variable).

$$\mathrm{z} = \frac{\mathrm{x} - \mu}{\sigma}$$

We substitute the given values.

$$z = \frac{40 - 40}{10} = 0$$

We obtained 0. If the Z score of x is zero, then the value of x is equal to the mean. In our case we have mean equal to 40%. So we can write that the number of candidates which will pass if 40% is fixed as the minimum marks for passing will be equal to $500 \cdot 0.4 = 200$ candidates.

Finally we solve the last task. We need to find the minimum percentage of marks for passing so that 350 candidates pass.

If it is known that 350 candidates are to pass then the probability of passing will be equal $\frac{350}{500} = 0.70$.

If we have z_1 is the minimum cut of mark then we can note the following.

$$\mathrm{P}(\mathrm{z} \geq \mathrm{z}_1) = 0.7 = 0.5 + 0.2 = P(0 < z < \infty) + P(0 < z < 0.525) = P(z \geq -0.525)$$

From the formula above we can write the following.

$$z_1 = -0.525$$

Then we can substitute into the formula.

$$z_1 = \frac{\mathrm{x} - \mu}{\sigma}$$

$$-0.525 = \frac{\mathrm{x} - 40}{10}$$

Now we can simplify the obtained equation by multiplying on 10 both sides of the equation.

$$\mathrm{x} - 40 = -5.25$$

Add 40 to both sides of the equation.

$$\mathrm{x} = -5.25 + 40$$

$$\mathrm{x} = 34.75 \approx 35\%$$

Finally we can note that 35% minimum pass marks could enable 350 candidates to pass out of 500 candidates.