Answer on Question #45523-Engineering-SolidWorks-CosmoWorks-Ansys

Let A and B be independent events with $P\ (A) = \frac{1}{4}$ and $P\ (A\ U\ B)\ =\ 2P\ (B)\ -\ P\ (A)$.

Find (a). P (B); (b).P (A|B); and (c). P (Bc|A).

Solution

(a). The probability of the union of A and B

$$P(A\ U\ B) = P(A) + P(B) - P(A\ \cap\ B).$$

Since A and B are independent:

$$P(A \cap B) = P(A) \cdot P(B).$$

And we have:

$$P(A\ U\ B) = P(A) + P(B) - P(A) \cdot P(B) = 2P(B) - P(A).$$

$$\frac{1}{4} + P(B) - \frac{1}{4} \cdot P(B) = 2P(B) - \frac{1}{4}.$$

$$P(B) = \frac{2}{5}.$$

(b).

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = P(A) = \frac{1}{4}.$$

(c). Since A and B are independent A and B^c are also independent. That's why

$$P(B^c|A) = \frac{P(B^c \cap A)}{P(A)} = \frac{P(B^c) \cdot P(A)}{P(A)} = P(B^c) = 1 - P(B) = 1 - \frac{2}{5} = \frac{3}{5}.$$