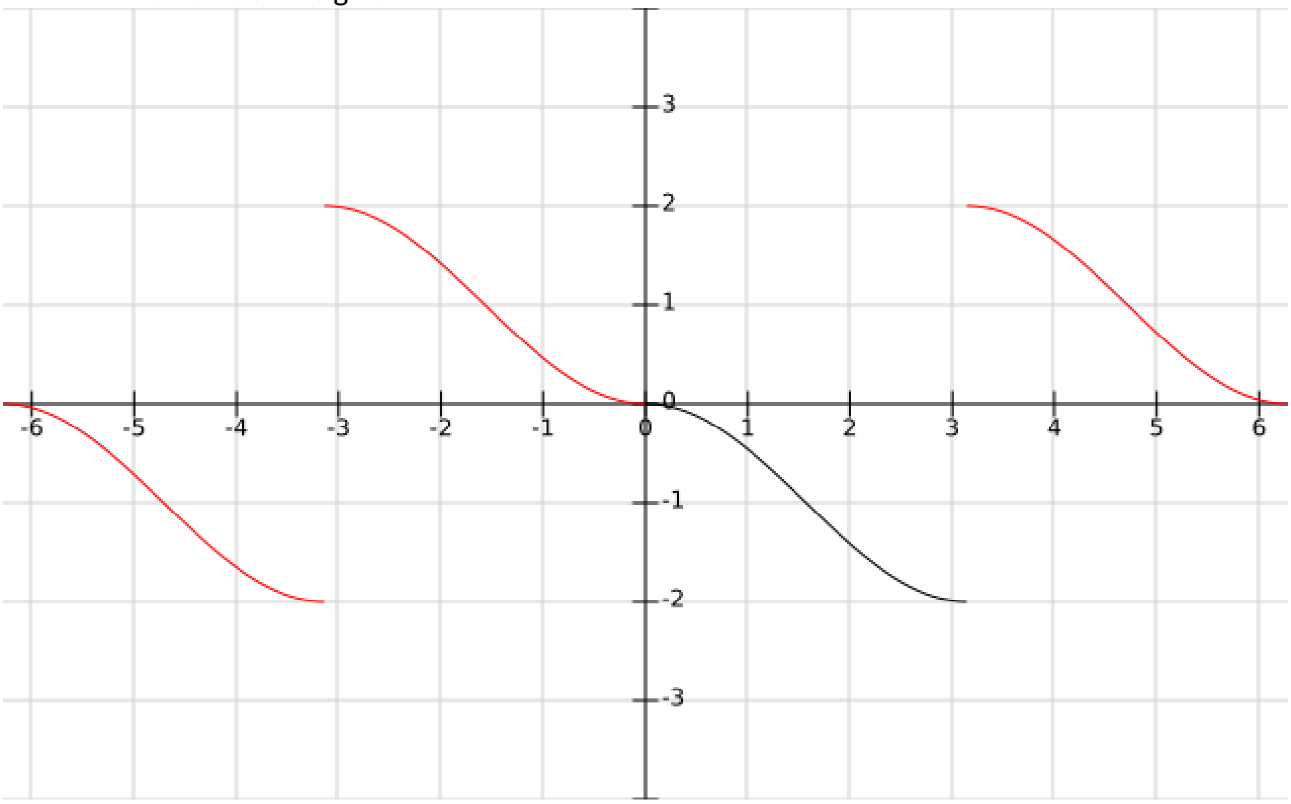
Answer on Question #44917, Engineering, SolidWorks | CosmoWorks | Ansys

Problem.

Find f(x)=cos x as a half range Fourier sine series in the range 0<=x<=pi and sketch the function within and outside of the given



range.

Solution.

Let $g(x) = f(x) - 1$ (we need to do this transformation to construct odd function).

We first extend $g(x)$ as an odd periodic function $G(x)$ of period 2π:

$$G(x) = \begin{cases} 1 - \cos x, & -\pi < x < 0 \\ \cos x - 1, & 0 \le x \le \pi \end{cases}$$

Since $G(x)$ is odd, then $a_n = 0$, for $n \ge 0$. We turn our attention to the coefficients b_n. For any $n \ge 1$, we have

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} G(x)\sin(nx)\,dx = \frac{1}{\pi}\int_0^{\pi} 2(\cos x - 1)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} (\sin((n+1)x) + \sin((n-1)x) - 2\sin nx)dx$$

Hence

$$b_1 = \frac{1}{\pi}\int_0^{\pi} (\sin 2x - 2\sin x)dx = \left(-\frac{1}{2\pi}\cos 2x + \frac{2}{\pi}\cos x\right)|_0^{\pi} = -\frac{4}{\pi},$$

$$b_{2n+1} = \frac{1}{\pi}\int_0^{\pi} (\sin((2n+2)x) + \sin(2nx) - 2\sin((2n-1)x))dx$$

$$= \left(-\frac{1}{2\pi(n+1)}\cos(2(n+1)x) - \frac{1}{2\pi n}\cos(2nx)\right)|_0^{\pi}$$

$$+ \frac{2}{\pi(2n+1)}\cos((2n+1)x)|_0^{\pi} = -\frac{4}{\pi(2n+1)},$$

$$b_{2n} = \frac{1}{\pi}\int_0^{\pi} (\sin((2n+1)x) + \sin((2n-1)x) - 2\sin(2nx))dx$$

$$= \left(-\frac{1}{\pi(2n+1)}\cos((2n+1)x) - \frac{1}{\pi(2n-1)}\cos((2n-1)x)\right)\Big|_0^{\pi}$$

$$+ \frac{2}{2\pi n}\cos(2nx)\Big|_0^{\pi} = \frac{2}{\pi(2n+1)} + \frac{2}{\pi(2n-1)} = \frac{8n}{\pi(2n+1)(2n-1)}$$

for all positive integer n.

Therefore

$$G(x) \sim \frac{4}{\pi}\left(-\sin x + \frac{2\sin 2x}{1\cdot 3} - \frac{\sin 3x}{3} + \frac{2\sin 4x}{3\cdot 5} + \cdots\right).$$

Then

$$f(x) \sim 1 + \frac{4}{\pi}\left(-\sin x + \frac{2\sin 2x}{1\cdot 3} - \frac{\sin 3x}{3} + \frac{4\sin 4x}{3\cdot 5} + \cdots\right)$$

Answer:

$$f(x) \sim -1 + \frac{2}{\pi}\left(-\sin x + \frac{4\sin 2x}{1\cdot 3} - \frac{\sin 3x}{3} + \frac{8\sin 4x}{3\cdot 5} + \cdots\right).$$