Answer on Question # 43804, Engineering, SolidWorks | CosmoWorks | Ansys

Task: $\cos^8\alpha - \sin^8\alpha = \frac{1}{4}\cos 2\alpha \cdot (3+\cos 4\alpha)$

Solution:

$$\cos^8\alpha - \sin^8\alpha = (\cos^4\alpha - \sin^4\alpha)(\cos^4\alpha + \sin^4\alpha)$$

$$\cos^4\alpha + \sin^4\alpha = (\cos^2\alpha)^2 + (\sin^2\alpha)^2 + 2\sin^2\alpha\cos^2\alpha - 2\sin^2\alpha\cos^2\alpha =$$

$$= (\cos^2\alpha + \sin^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha = 1 - 2\sin^2\alpha\cos^2\alpha = 1 - \frac{\sin^2 2\alpha}{2};$$

$$\cos^4\alpha - \sin^4\alpha = (\cos^2\alpha - \sin^2\alpha)(\cos^2\alpha + \sin^2\alpha) = \cos 2\alpha \Rightarrow$$

$$\cos^8\alpha - \sin^8\alpha = \cos 2\alpha(1 - \frac{\sin^2 2\alpha}{2})$$

$$1 - \frac{\sin^2 2\alpha}{2} = 4(\frac{1}{4} - \frac{2\sin^2 2\alpha}{16}) = 4(\frac{4 - 2\sin^2 2\alpha}{16}) = \frac{3 + 1 - 2\sin^2 2\alpha}{4} = \frac{3 + \cos 4\alpha}{4} \Rightarrow$$

$$\cos^8\alpha - \sin^8\alpha = \cos 2\alpha \cdot \frac{3 + \cos 4\alpha}{4} = \frac{1}{4}\cos 2\alpha \cdot (3 + \cos 4\alpha)$$

So, equality is proved.