

Answer on Question #43728-Engineering-Other

Prove that $4 \sin 3a \cos^3 a + 4 \cos 3a \sin^3 a = 3 \sin 4a$

Solution

We know that

$$\begin{aligned}\sin 2a &= 2 \sin a \cos a, & \cos 2a &= \cos^2 a - \sin^2 a \\ \sin 3a &= 3 \sin a - 4 \sin^3 a, & \cos 3a &= 4 \cos^3 a - 3 \cos a.\end{aligned}$$

So

$$\begin{aligned}4 \sin 3a \cos^3 a + 4 \cos 3a \sin^3 a &= 4(3 \sin a - 4 \sin^3 a) \cos^3 a + 4(4 \cos^3 a - 3 \cos a) \sin^3 a \\ &= 12 \sin a \cos^3 a - 16 \sin^3 a \cos^3 a + 16 \sin^3 a \cos^3 a - 12 \cos a \sin^3 a \\ &= 12(\sin a \cos^3 a - \cos a \sin^3 a) = 6 \cdot (2 \sin a \cos a)(\cos^2 a - \sin^2 a) = 6 \sin 2a \cos 2a \\ &= 3 \cdot (2 \sin 2a \cos 2a) = 3 \sin 4a.\end{aligned}$$