## Answer on Question \#43469-Engineering-Other

Answer the following questions for the function $f(x)=\frac{\left(x^{3}\right)}{x^{2}-9}$ defined on the interval [-17,15]. Enter points, such as inflection points in ascending order, i.e. smallest $x$ values first.
A. The function $f(x)$ has vertical asymptotes? (2 of them)
B. $f(x)$ is concave up on the region () to () and () to ().
C. The inflection point for this function is?

## Solution

A. The function has vertical asymptotes if it is reduced (no common factors in numerator and denominator) and the denominator is zero.
Thus $x^{2}-9 \rightarrow x= \pm 3$. The vertical asymptotes are at $x=-3$ and $x=3$.
B. The function is concave up on an interval where the 2 nd derivative is positive.

$$
\begin{gathered}
f(x)=\frac{\left(x^{3}\right)}{x^{2}-9} \\
f^{\prime \prime}(x)=\frac{\left(x^{4}-18 x^{2}+81\right) \cdot\left(4 x^{3}-27 \cdot 2 x\right)-\left(x^{4}-27 x^{2}\right)\left(4 x^{3}-36 x\right)}{\left(x^{2}-9\right)^{4}} \\
=\frac{18 x^{5}+324 x^{3}-4374 x}{\left(x^{2}-9\right)^{4}}=\frac{18\left(x^{5}+18 x^{3}-243 x\right)}{\left(x^{2}-9\right)^{4}}=\frac{18 x\left(x^{2}-9\right)\left(x^{2}+27\right)}{\left(x^{2}-9\right)^{4}} \\
=\frac{18 x\left(x^{2}+27\right)}{\left(x^{2}-9\right)^{3}} .
\end{gathered}
$$

$f^{\prime \prime}(x)>0$ on $(-3,0)$ and $(3, \infty)$ so the given function is concave up on $(-3,0)$ and $(3,15)$.
C. The inflection point occurs when $f^{\prime \prime}(x)=0$ and the sign of the second derivative changes parity. The second derivative is zero at $x=0$ which is the inflection point.

