Answer on Question #41475, Engineering Other

Sand is leaking from the back of a dump truck and forming a conical pile on the ground. The sand is leaking at the rate of 0.2 m3 per hour. If the base radius of the pile is always 0.4 times the height, how fast is the base radius changing when the height is 1.5 m? Show all the working.

(Volume of cone of height h and base radius r is V = 1πr2h.)

**Solution**

Volume of cone of height h and base radius r is

$$\mathrm{V} = \frac{1}{3}\pi \mathrm{r}^2\mathrm{h}.$$

The base radius of the pile is always 0.4 times the height:

$$r = 0.4h \rightarrow h = \frac{r}{0.4} = 2.5r,$$

Volume of our cone is

$$\mathrm{V} = \frac{1}{3}\pi \mathrm{r}^2 \cdot 2.5r = 2.5 \cdot \frac{1}{3}\pi \mathrm{r}^3.$$

Volume flow rate is

$$\frac{dV}{dt} = \frac{d}{dt}\left(2.5 \cdot \frac{1}{3}\pi \mathrm{r}^3\right) = 2.5\frac{d}{dt}\left(\frac{1}{3}\pi \mathrm{r}^3\right) = 2.5\pi \mathrm{r}^2\frac{dr}{dt} = 0.2\frac{\mathrm{m}^3}{\mathrm{hour}}.$$

When the height is 1.5 m base radius of the pile is $r_1 = 0.4 \cdot 1.5\ \mathrm{m} = 0.6\ \mathrm{m}$.

The rate of change of base radius is

$$\frac{dr}{dt} = \frac{1}{2.5\pi \mathrm{r}^2}\frac{dV}{dt} = \frac{1}{2.5\pi \cdot 0.6^2} \cdot 0.2\frac{\mathrm{m}}{\mathrm{hour}} = \frac{2}{9\pi}\frac{\mathrm{m}}{\mathrm{hour}} \approx 0.07\frac{\mathrm{m}}{\mathrm{hour}}.$$

**Answer:** $\mathbf{0.07\frac{m}{hour}}$.