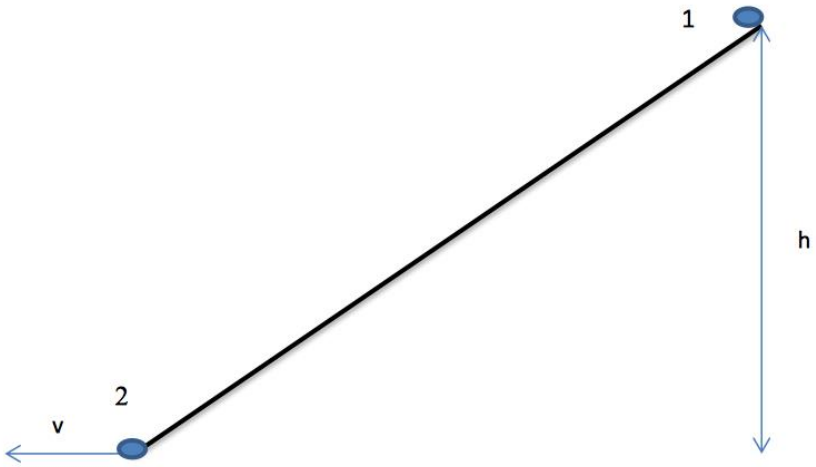
Answer on Question#40731 – Physics - Mechanics | Kinematics | Dynamics

Suppose you are designing a cart for carrying goods downhill. To maximize the cart speed, should you design the wheels so that their moments of inertia about their rotation axes are large or small, or it does not matter? Explain assuming that the mechanical energy is conserved.

Solution:



$m-$ mass of the cart;
$v-$ speed of the cart;
$I-$ sum of moments of inertia all wheels;
$\omega-$ angular velocity of wheels;

The conservation law of mechanical energy:
$T+U=const$
or:
$T_1+U_1=T_2+U_2$
For state (1): $T_1=0; U_1=mgh;$
For state (2): $U_2=0; T_2=\frac{mv^2}{2}+\frac{I\omega^2}{2};$
Formula for the angular velocity (r-radius of wheel):

$$\omega=\frac{v}{r}$$

Therefore:

$$mgh=\frac{mv^2}{2}+\frac{I\left(\frac{\omega}{r}\right)^2}{2}=\frac{mv^2}{2}\left(1+\frac{I}{mr^2}\right)$$

$$v=\sqrt{\frac{2gh}{\left(1+\frac{I}{mr^2}\right)}}$$

Obviously, to maximize the cart speed, we should design the wheels so that their moments of inertia about their rotation axes are small as it possible. In ideal case $I\ll mr^2$ and:

$$v_{max} = \sqrt{2gh}$$

Answer: we should design the wheels so that their moments of inertia about their rotation axes are small as it possible