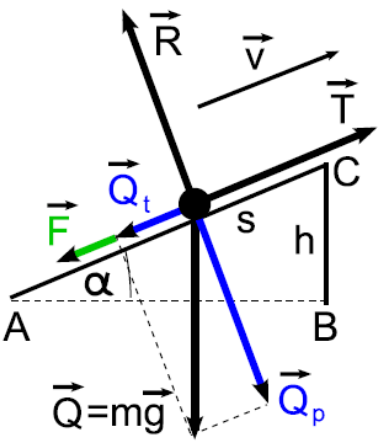
A vehicle of mass 700 kg accelerates uniformly from rest to a velocity of 60 kmh−1 in 10 s whilst ascending a 15% gradient. The frictional resistance to motion is 0.5 kN. Making use of D'Alembert's principle, determine:

i) the tractive effort between the wheels and the road surface

ii) the work done in ascending the slope

iii) the average power developed by the engine

Solution



The picture shows forces acting on the vehicle. There are: the gravitational force $\vec{Q} = m\vec{g}$, the reaction of road's surface $\vec{R}$ and frictional force $\vec{F}$,working against the vehicle's velocity $\vec{v}$. The problem's text claims that the vehicle is ascending so the vectors $\vec{v}$ and $\vec{F}$ have directions as in the picture. The vehicle is moved uphill by the tractive force $\vec{T}$ which does the real work. The force $\vec{Q}$ can be split into 2 compounds: $\overrightarrow{Q_p}$, perpendicular to the road surface and $\overrightarrow{Q_t}$ parallel to the road. The vector sum of $\overrightarrow{Q_p} + \vec{R}$ gives zero but $\overrightarrow{Q_p}$ is that force which causes the frictional resistance.

Using a vector notation one may write the Newton's second law of dynamics for the vehicle as follows:

$$m\vec{a} = \vec{T} + \overrightarrow{Q_t} + \vec{F} \qquad (1)$$

Because vectors $\vec{F}$ and $\overrightarrow{Q_t}$ are opposite to $\vec{T}$ one should take their values with a minus sign in the next equation. In addition because the angle between $\vec{Q}$ and $\overrightarrow{Q_p}$ equals α (the same as the slope of the road) the value of $\overrightarrow{Q_t}$ may be written as mg sin(α). Therefore:

$$m\vec{a} = T - mg\sin\alpha - F \qquad (2)$$

From the equation (2) one may calculate the value of the tractive effort T. The acceleration a may be calculated as $a = \frac{v}{t}$, where v is the given velocity, $(v = 100\ km/h)$ and t is the acceleration time $(t = 14\ s)$. From eq. (2), substituting a one obtains:

$$T = m\frac{v}{t} + F + mg\sin\alpha \qquad (3)$$

Let put numeric values into equation (3). The velocity must be expressed in $\frac{m}{s}$ (by dividing it by 3.6). Sinus α is the "gradient", equals 0.15.

$$T = 700 * \frac{\frac{60}{3.6}}{10} + 500 + 700 * 10 * 0.15 = 2.7 \text{ kN}.$$

The work W done in ascending the slope equals T times s, where s is the slope length. (The vector $\vec{T}$ is parallel the road so $\cos(\varphi)$ equals 1 in the formula for mechanical work). The length s can be calculated from equation (5)

$$s = \frac{1}{2}at^2 = \frac{1}{2}\frac{v}{t}t^2 = \frac{v}{2}t \qquad (5)$$

The last formula above on the right side shows that in a uniformly accelerating movement the distance s can be calculated by multiplying time by the average velocity.

Therefore the amount of work W equals:

$$W = Fs = \frac{Tvt}{2} \qquad (6)$$

Using the value of F from eq. (4) the above formula gives:

$$W = \frac{1}{2} * 2.7 * 10^3 * \left(\frac{60}{3.6}\right) * 10 = 225\ kJ. \qquad (7)$$

The average power P equals W divided by time t, therefore:

$$P = \frac{W}{t} = \frac{225kJ}{10s} = 22.5\ kW.$$

Answer: $\mathbf{2.7}$ kN; $\mathbf{225}\ \boldsymbol{kJ}$; $\mathbf{22.5}\ \boldsymbol{kW}$.