Answer on Question #85198 - Economics – Macroeconomics

Question:

Start from the Cobb-Douglas production function where the parameters á and â are constant, 0 < á < 1 and á + â = 1.2. Derive the expression for the growth rate of output per worker in balanced growth equilibrium. Show all your steps. In this case, will the growth rate of output per worker be equal to, less than, or greater than the growth rate of efficiency in balanced growth equilibrium?

Answer

The Cobb-Douglas production function in given case is following:

$$Y = AK^{á}L^{1.2-á}$$

The growth rate in any point of time t is defined as:

$$G_t^Y = \frac{1}{Y_t}\frac{dY_t}{dt}$$

In this case the growth rate is the function of the growth rates of labor, capital and technology by the differentiating the right-hand-side of equation with respect to time. Therefore, we will get:

$$\frac{dY_t}{dt} = \frac{dA_tK_t^{á}L_t^{1.2-á}}{dt} = K_t^{á}L_t^{1.2-á}\frac{dA_t}{dt} + áA_tL_t^{1.2-á}\frac{dK_t^{á}}{dt} + (1.2-á)A_tK_t^{á}\frac{dL_t^{1.2-á}}{dt}$$

The impact of changes in capital and labor:

$$\frac{dK_t^{á}}{dt} = \frac{dK_t^{á}}{dK_t}\frac{dK_t}{dt} = á\ \mathrm{K}_{\mathrm{t}}^{á-1.2}\frac{dK_t}{dt}$$

$$\frac{dL_t^{1.2-á}}{dt} = \frac{dL_t^{1.2-á}}{dL_t}\frac{dL_t}{dt} = (1.2-á)\ \mathrm{L}_{\mathrm{t}}^{-á}\frac{dL_t}{dt}$$

The growth rate of output is calculated by dividing both sides by Y_t or $AK^{á}L^{1.2-á}$:

$$\frac{1}{Y_t}\frac{dY_t}{dt} = \frac{K_t^{á}L_t^{1.2-á}}{A_tK^{á}L^{1.2-á}}\frac{dA_t}{dt} + á\frac{A_tL_t^{1.2-á}}{A_tK^{á}L^{1.2-á}}\frac{dK_t}{dt} + (1.2-á)\frac{A_tK_t^{á}}{A_tK^{á}L^{1.2-á}}\frac{dL_t}{dt}$$

Then we get:

$$\frac{1}{Y_t}\frac{dY_t}{dt} = \frac{1}{A_t}\frac{dA_t}{dt} + á\frac{1}{K^{á}}\frac{dK_t^{á}}{dt} + \frac{1}{L_t}\frac{dL_t}{dt}$$

It can be written in more intuitive form as:

$$G_t^Y = G_t^A + áG_t^K + (1.2-á)G_t^L$$

Then the growth rate per worker is the growth rate of output minus the growth in the number of workers:

$$G_t^Y - 1.2G_t^L = G_t^A + \text{á}(G_t^K - G_t^L)$$

Since 0 < á < 1, the growth rate per worker is greater than the growth rate of efficiency.