Question #76823, Economics, Microeconomics

John has a utility function U(B,Z)=AB^(1/α) Z^(1/β), where A, and are constants, B is burritos, and Z is pizzas. If the price of burritos, P b is $10 and the price of pizzas, P z , is $5, and Y is $1790, what is John's optimal bundle?

$$U(B,Z) = AB^{\frac{1}{\alpha}}Z^{\frac{1}{\beta}}$$

$$I = p_B * B + p_Z * Z$$

$$1790 = 10 * B + 5 * Z$$

$$MU_b = A\frac{1}{\alpha}B^{\frac{1}{\alpha}-1}Z^{\frac{1}{\beta}}$$

$$MU_z = A\frac{1}{\beta}B^{\frac{1}{\alpha}}Z^{\frac{1}{\beta}-1}$$

$$MRS_{b,z} = -\frac{\frac{1}{\alpha}B^{\frac{1}{\alpha}-1}Z^{\frac{1}{\beta}}}{\frac{1}{\beta}B^{\frac{1}{\alpha}}Z^{\frac{1}{\beta}-1}} = -\frac{\beta Z}{\alpha B}$$

$$MRS_{b,z} = -\frac{p_b}{p_z}$$

$$\frac{p_b}{p_z} = \frac{\beta Z}{\alpha B}$$

For Z*:

$$p_b B = \frac{\beta}{\alpha}p_z Z$$

From the budget constraint, we have the following:

$$\frac{\beta}{\alpha}p_z Z + p_z Z = I$$
$$\frac{\beta}{\alpha} * 5Z + 5Z = 1790$$
$$5Z(\frac{\beta}{\alpha} + 1) = 1790$$
$$\frac{\beta + \alpha}{\alpha} * 5Z = 1790$$
$$Z = \frac{1790}{5} * \frac{\alpha}{\alpha + \beta} = \frac{358\alpha}{\alpha + \beta}$$

For B*:

$$p_b B + p_z \frac{358\alpha}{\alpha + \beta} = 1790$$

$$10B + 5\frac{358\alpha}{\alpha+\beta} = 1790$$

$$10B + 1790\frac{\alpha}{\alpha+\beta} = 1790$$

$$10B = 1790(1 - \frac{\alpha}{\alpha+\beta})$$

$$B = 179(\frac{\alpha+\beta-\alpha}{\alpha+\beta}) = \frac{179\beta}{\alpha+\beta}$$

Optimal bundle is B=$\frac{179\beta}{\alpha+\beta}$ and Z=$\frac{358\alpha}{\alpha+\beta}$