Answer on Question #73879 -Economics - Microeconomics

Given the production function $Q = 30K^{0.7} L^{0.5}$ and input prices r = 20 and w = 30.

(a) Determine an equation for the expansion path

(b) What is the efficient input combination for an output rate of Q = 200? For 500?

Answer.

a)

Expansion path is determined by the condition

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

And,

$$MP_L = (30K^{0.7}L^{0.5})' = 30 \times 0.5 \times K^{0.7}L^{-0.5} = \frac{15K^{0.7}}{L^{0.5}}$$

$$MP_K = (30K^{0.7}L^{0.5})' = 30 \times 0.7 \times K^{-0.3}L^{0.5} = \frac{21L^{0.5}}{K^{0.3}}$$

So,

$$\frac{\frac{15K^{0.7}}{L^{0.5}}}{\frac{21L^{0.5}}{K^{0.3}}} = \frac{30}{20}$$

$$\frac{15K^{0.7} \times K^{0.3}}{L^{0.5} \times 21L^{0.5}} = \frac{30}{20}$$

$$\frac{5K}{7L} = \frac{3}{2}$$

$$10K = 21L$$

The equation of expansion path is

$$10K - 21L = 0$$

b)

We have to solve the system of equations

$$30K^{0.7}L^{0.5} = 200$$
$$10K = 21L$$

And, if K=2.1L then

$$30 \times (2.1L)^{0.7}L^{0.5} = 200$$
$$30 \times (2.1L)^{0.7}L^{0.5} = 200$$
$$50.43L^{1.2} = 200$$
$$L = 3.15$$
$$K = 2.1 \times 3.15 = 6.62$$

The efficient input combination for an output rate of Q = 200 is
L=3.15
K=6.62

The efficient input combination for an output rate of Q = 500 is calculated from

$$50.43L^{1.2} = 500$$
$$L = 6.76$$
$$K = 2.1 \times 3.15 = 14.2$$