Question #73327

(a) Derive the IS relation. (Hint: You want an equation with Y on the left side and everything else on the right.)

(b) Derive the LM relation. (Hint: It will be convenient for later use to rewrite this equation with i on the left side and everything else on the right.)

Part II: Solve for equilibrium

(c) Solve for the equilibrium real output. (Hint: Substitute the expression for the interest rate given by the LM equation into the IS equation and solve for output.)

(d) Solve for equilibrium interest rate. (Hint: Substitute the value you obtained for Y in part (c) into either IS or LM equations and solve for i. If your algebra is correct you should get the same answer from both equations)

(e) Solve for the equilibrium values of C and I, and verify the value you obtained for Y by adding C, I and G.

ANSWERS

a) The IS curve can be represented according to output with help of the following equation

$$Y = \frac{a+e+g}{1-b(1-t)+m\prime} + \frac{1}{1-b(1-t)+m\prime} \cdot G - \frac{b}{1-b(1-t)+m'} \cdot Ta - \frac{d+n}{1-b(1-t)+m'} \cdot R$$

where

Y is output or income**. G** is government expenditures or consumption, **Ta** describes autonomous taxes, **R (i)** is interest rate, **a** is autonomous consumption, **e** is autonomous investition, **g** represents the autonomous net export, **t** is for tax rate, **b** is MPC (Marginal propensity to consume), $\mathbf{m'} -$ represents MPI (Marginal propensity to import), **d** is the empirical coefficient which describes the sensitivity of investitions to the changes of interest rate, and eventually **n** is like **d,** it is an empirical coefficient which shows sensitivity of net export in response to changes of interest rate.

b) The LM curve can be described according to interest rate (R or i) with the representation of the following equation - $R = \frac{k}{h} \cdot Y - \frac{1}{h} \cdot \frac{M}{P}$, where **M/P** describes real demand for money, **k** and **h** show the sensitivity of real money demand in response to changes of output (**k**) and interest rate (**h**).

c) The equilibrium output (Y*) can be found from the following equation.

$$Y^* = \frac{a + e + g + G - b \cdot Ta}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')} \cdot h + \frac{d + n}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')} \cdot \frac{M}{P}$$

d) The equilibrium level of interest rate (R*or i*) can be found from the following equation.

$$R^* = \frac{a + e + g + G - b \cdot Ta}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')} \cdot k - \frac{1 - b \cdot (1 - t) + m'}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')} \cdot \frac{M}{P}$$

e) To find the equilibrium value of C (C*), we should use the following two equations

C=a+b(Y-T), T=Ta+t·Y and the equilibrium output we have got above. After some calculations we will got

$$C^* = \frac{(a - b \cdot Ta) \cdot (k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m'))}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')} + \frac{b \cdot (1 - t)(d + n)}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')} \cdot \frac{M}{P}$$

Same process can be performed for I, but in this case for I we will use the following equation $I = e - d \cdot R$. In this case the equilibrium level of I (I*) can be find from the following equation

$$I^* = e - \frac{d \cdot (a + e + g + G - b \cdot Ta)}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')} \cdot k - \frac{d \cdot (1 - b \cdot (1 - t) + m')}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')} \cdot \frac{M}{P}$$

According to above represented equations of C*, I* now we can find Y just by using the main equation of macroeconomics Y=C+I+G+NX, so lets put the values of I*, C* to the main equation in this case we will have the following equation for equilibrium Y

$$Y = C_0 + I_0 + G + \frac{b \cdot (1 - t)(d + n) - d \cdot (1 - b \cdot (1 - t) + m')}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')} \cdot \frac{M}{P}$$

where

$C_0 = \frac{(a - b \cdot Ta) \cdot (k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m'))}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')}$, $I_0 = e - \frac{d \cdot (a + e + g + G - b \cdot Ta)}{k \cdot (d + n) + h \cdot (1 - b \cdot (1 - t) + m')} \cdot k$