Question #62852 - Economics – Macroeconomics | Completed

Question

The production function $Y = A*K^{\alpha}*L^{\beta}$ with A = 2, α = 0.4 and β = 0.6 exhibits the following properties:

(a) concavity in L

(b) convexity in L

(c) constant returns to scale in K and L

(d) homogeneity of degree 1 (in K and L)

(e) increasing returns in K and L

Answer

(a) Hessian is a negative-semidefinite matrix at all points ($d^2Y/dL^2<0$, $d^2K/dK^2<0$)=>

=> $Y=2K^{0.4}L^{0.6}$=>$dY/dK=0.8K^{-0.6}L^{0.6}$=> $d^2Y/dL^2=-0.48\ K^{-1.6}L^{0.6}<0$

$dY/dL=1.2K^{0.4}L^{-0.4}$=> $d^2Y/dL^2=-0.48\ K^{0.4}L^{-1.4}<0$ – concave function

(b) dY/dK and dY/dL > 0, d^2Y/dK^2 and $d^2Y/dL^2<0$ – without convestity

(c) $Y = A*K^{\alpha}*L^{\beta}$ => $A*(t*K)^{\alpha}*(t*L)^{\beta}= A*t^{\alpha}*K^{\alpha}*t^{\beta}*L^{\beta}= A*t^{\alpha+\beta}*K^{\alpha}*L^{\beta}= t^{(\alpha+\beta)}\ *Y$, (α+β)=0.4+0.6=1 – constant returns to scale (if (α+β)>1 – increasing returns to scale, if (α+β)<1 – decreasing returns to scale);

(d) $Y = A*K^{\alpha}*L^{\beta}$ => $A*(t*K)^{\alpha}*(t*L)^{\beta}= A*t^{\alpha}*K^{\alpha}*t^{\beta}*L^{\beta}= A*t^{\alpha+\beta}*K^{\alpha}*L^{\beta}= t^{(\alpha+\beta)}\ *Y$, α+β=0.4+0.6=1 – homogeneity of degree 1;

(e) $Y = 2*K^{0.4}*L^{0.6}$, $dY/dK=2*0.4\ *(L/K)^{0.6}=0.8*(L/K)^{0.6}>0$ – when K increases, the total output will increase (marginal product is positive), the same for L: $dY/dL=2*0.6\ *(K/L)^{0.4}=1.2*(K/L)^{0.4}>0$