

Answer on Question 50825, Economics, Microeconomics

Question:

For each of the following production functions, determine whether returns scales are decreasing, constant or increasing:

a. $Q = 2K + 3L + KL$

b. $Q = 20K^{0.6}L^{0.5}$

c. $Q = 100 + 3K + 2L$

d. $Q = 5K^aL^b$, where $a + b = 1$

e. $Q = 10K^aL^b$, where $a + b = 1.2$

f. $Q = \frac{K}{L}$

Answer:

Basically, the returns to scale refers to how much output changes given a proportional change in all inputs, where all the inputs change by the same factor. For example:

$$F(zK, zL) = AK^aL^b \begin{cases} < zF(K, L) \rightarrow \text{Decreasing Returns to Scale} \\ = zF(K, L) \rightarrow \text{Constant Returns to Scale} \\ > zF(K, L) \rightarrow \text{Increasing Returns to Scale} \end{cases}$$

a. Let $Q_0 = F(K, L) = 2K + 3L + KL$ be the initial production function. Let us multiply it by factor z and call it Q_1 :

$$Q_1 = F(zK, zL) = 2(zK) + 3(zL) + (zK)(zL) = z(2K + 3L + zKL)$$

It is obvious that $F(zK, zL) > zF(K, L)$:

$$z(2K + 3L + zKL) > z(2K + 3L + KL)$$

$$2K + 3L + zKL > 2K + 3L + KL$$

$$zKL > KL$$

$$z > 1.$$

This production function represents increasing returns to scale.

b. Let $Q_0 = F(K, L) = 20K^{0.6}L^{0.5}$ be the initial production function, then after multiplying it by factor z we obtain:

$$Q_1 = F(zK, zL) = 20(zK)^{0.6}(zL)^{0.5} = z^{0.6}z^{0.5}20K^{0.6}L^{0.5} = z^{1.1}Q_0$$

In this case $F(zK, zL) > zF(K, L)$.

This production function represents increasing returns to scale.

c. Let $Q_0 = F(K, L) = 100 + 3K + 2L$ be the initial production function, then after multiplying it by factor z we obtain:

$$Q_1 = F(zK, zL) = 100 + 3(zK) + 2(zL)$$

In this case $F(zK, zL) < zF(K, L)$:

$$100 + 3(zK) + 2(zL) < 100z + 3(zK) + 2(zL)$$

This production function represents decreasing returns to scale.

d. Let $Q_0 = F(K, L) = 5K^aL^b$, where $a + b = 1$, be the initial production function, then after multiplying it by factor z we obtain:

$$Q_1 = F(zK, zL) = 5(zK)^a(zL)^b = z^a z^b 5K^a L^b = z^{a+b} Q_0 = z Q_0$$

In this case $F(zK, zL) = zF(K, L)$.

This production function represents constant returns to scale.

e. Let $Q_0 = F(K, L) = 10K^aL^b$, where $a + b = 1.2$, be the initial production function, then after multiplying it by factor z we obtain:

$$Q_1 = F(zK, zL) = 10(zK)^a(zL)^b = z^a z^b 10K^a L^b = z^{a+b} Q_0 = z^{1.2} Q_0$$

In this case $F(zK, zL) > zF(K, L)$.

This production function represents increasing returns to scale.

f. Let $Q_0 = F(K, L) = \frac{K}{L} = KL^{-1}$ be the initial production function, then after multiplying it by factor $z = 2$ we obtain:

$$Q_1 = F(zK, zL) = (2K)(2L)^{-1} = 2 \cdot 2^{-1} KL^{-1} = KL^{-1}$$

In this case $F(2K, 2L) = 2F(K, L)$.

This production function represents constant returns to scale.

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