

Answer on Question #50503, Economics, Microeconomics

(a) A consumer's indirect utility function  $v(p, w)$  gives the consumer's maximal utility when faced with a price level  $p$  and an amount of income  $w$ . It represents the consumer's preferences over market conditions. This function is called indirect because consumers usually think about their preferences in terms of what they consume rather than prices. A consumer's indirect utility  $v(p, w)$  can be computed from its utility function  $u(x)$  by first computing the most preferred bundle  $x(p, w)$  by solving the utility maximization problem; and second, computing the utility  $u(x(p, w))$  the consumer derives from that bundle. The indirect utility function for consumers is analogous to the profit function for firms.

(b) Roy's identity (named for French economist René Roy) is a major result in microeconomics having applications in consumer choice and the theory of the firm. The lemma relates the ordinary (Marshallian) demand function to the derivatives of the indirect utility function. Specifically, where  $V(P, Y)$  is the indirect utility function, then the Marshallian demand function for good  $i$  can be calculated as:

$$x_i^m = -\frac{\frac{\partial V}{\partial p_i}}{\frac{\partial V}{\partial Y}}$$

(c) Find the expenditure function of the consumer  $e(p, u)$  where price of  $x = 1$  and price of  $y = p$ . Formally, the expenditure function is defined as follows. Suppose the consumer has a utility function  $u$  defined on  $L$  commodities. Then the consumer's expenditure function gives the amount of money required to buy a package of commodities at given prices  $p$  that give utility of at least  $u^*$ ,

$$e(p, u^*) = \min_{x \in \mathcal{X} \text{ s.t. } u(x) \geq u^*} p \cdot x$$

(d) Find the Hicksian demand function  $h_y(p, u)$  for commodity  $y$ , where the price of  $x$  is 1 and the price of  $y$  is  $p$ .

A consumer's Hicksian demand correspondence is the demand of a consumer over a bundle of goods that minimizes their expenditure while delivering a fixed level of utility. If the correspondence is actually a function, it is referred to as the Hicksian demand function, or compensated demand function.

$$h(p, \bar{u}) = \arg \min_x \sum_i p_i x_i$$